Chapter-1 : Rational Numbers

Exercise-1

1. Find the value of the following :

(a) $\left|\frac{-6}{13}\right| = \frac{6}{13}$

(b) $\left|\frac{7}{13} + \frac{-9}{13}\right|$

Taking L.C.M.

$= \left|\frac{7-9}{13}\right| = \left|\frac{-2}{13}\right| = \frac{2}{13}$

(c) $\left|\frac{-14}{27}\right| - \left|\frac{-3}{20}\right| = \frac{14}{27} - \frac{3}{20}$

Taking L.C.M.

$$= \frac{20\times 14 - 27\times 3}{27\times 20}$$

$$= \frac{280-81}{540}$$

$$= \frac{199}{540}$$

2. Express the following rational numbers in the standard form :

(a) $\frac{65}{-39}$

Dividing numerator and denominator by 13,

$$= \frac{65\div 13}{-39\div 13}$$

So, the standard form of rational number

$$= \frac{5}{-3}$$

(b) $\frac{256}{216}$

Dividing numerator and denominator by 8,

$$= \frac{256\div 8}{216\div 8} = \frac{32}{27}$$

So, the standard form of ration number

$$= \frac{32}{27}$$

(c) $\frac{-72}{-96}$

Dividing numerator and denominator by 24,

$$= \frac{-72 \div 24}{-96 \div 24} = \frac{3}{4}$$

So, $\frac{3}{4}$ is the standard form.

(d) $\frac{13}{169}$

Dividing numerator and denominator by 13,

$$= \frac{13 \div 13}{169 \div 13} = \frac{1}{13}$$

So, $\frac{1}{13}$ is the standard form.

3. Arrange the following in ascending order :

$$\frac{9}{25}, \frac{2}{5}, \frac{14}{-75}, \frac{-19}{10}, \frac{8}{15}$$

L.C.M. of denominator, 25, 5, –75, 10, 15 is 150. Now, convert each of the given rational numbers into an equivalent rational number with denominator 150.

$$\frac{9}{25} = \frac{9 \times 6}{25 \times 6} = \frac{54}{150}$$

$$\frac{2}{5} = \frac{2 \times 30}{5 \times 30} = \frac{60}{150}$$

$$\frac{14}{-75} = \frac{14 \times 2}{-75 \times 2} = \frac{28}{-150}$$

$$\frac{-19}{10} = \frac{19 \times 15}{10 \times 15} = \frac{-285}{150}$$

$$\frac{8}{15} = \frac{8 \times 10}{15 \times 10} = \frac{80}{150}$$

The equivalent rational numbers are

$$\frac{54}{150}, \frac{60}{150}, \frac{28}{-150}, \frac{-285}{150}, \frac{80}{150}$$

Arranging the equivalent rational number in ascending order,

$$\frac{-285}{150} < \frac{28}{-150} < \frac{54}{150} < \frac{60}{150} < \frac{80}{150}$$

Thus the given numbers in ascending order are

$$\frac{-19}{10} < \frac{14}{-75} < \frac{9}{25} < \frac{2}{5} < \frac{8}{15}$$

4. Arrange the following in descending order :

$$\frac{6}{11}, \frac{-4}{5}, \frac{10}{13}, \frac{-17}{23}, \frac{-5}{11}$$

L.C.M. of denominators 11, 5, 13, 23 is 16445. now convert each of the given rational numbers into an equivalent rational number with denominator 16445.

$$\frac{6}{11}=\frac{6\times1495}{11\times1495}=\frac{8970}{16445}$$

$$\frac{-4}{5}=\frac{-4\times3289}{5\times3289}=\frac{-13156}{16445}$$

$$\frac{10}{13}=\frac{10\times1265}{13\times1265}=\frac{12650}{16445}$$

$$\frac{-17}{23}=\frac{17\times715}{23\times715}=\frac{-12155}{16445}$$

$$\frac{-5}{11}=\frac{-5\times1495}{11\times1495}=\frac{-7475}{16445}$$

The equivalent rational numbers are

$$\frac{8970}{16445},\frac{-13156}{16445},\frac{12650}{16445},\frac{-12155}{16445},\frac{-7475}{16445}$$

Arranging the equivalent rational numbers in descending order :

$$\frac{12650}{16445}>\frac{8970}{16445}>\frac{-7475}{16445}>\frac{-12155}{16445}>\frac{-13156}{16445}$$

Thus the given number in descending order

$$\frac{10}{13}>\frac{6}{11}>\frac{-5}{11}>\frac{-17}{23}>\frac{-4}{5}$$

5. Find

(a) $\frac{-2}{15}+\frac{7}{30}$

L.C.M. of 15 and 30 is 30

$$=\frac{-2\times2+7\times1}{30}=\frac{-4+7}{30}$$

$$=\frac{3}{30}=\frac{1}{10}$$

(b) $\frac{-8}{13}+\left(\frac{-12}{13}\right)$

$$=\frac{-8}{13}-\frac{12}{13}$$

L.C.M. of 13 and 13 is 13

$$=\frac{-8-12}{13}=\frac{-20}{13}$$

(c) $\frac{21}{25}-\frac{16}{25}+\frac{7}{25}$

L.C.M. is 25

$$\frac{21-16+7}{25}=\frac{28-16}{25}=\frac{12}{25}$$

(d) $\frac{-6}{13} - \left(\frac{-5}{15}\right)$

$$= \frac{-6}{13} + \frac{5}{15}$$

L.C.M. of 13 and 15 is 195.

$$= \frac{-6 \times 15 + 13 \times 5}{195} = \frac{-90 + 65}{195} = \frac{-25}{195}$$

Divide the numirator and denominator by 5.

$$= \frac{-25 \div 5}{195 \div 5} = \frac{-5}{39}$$

(e) $\frac{3}{11} \times \frac{2}{5}$

$$= \frac{3 \times 2}{11 \times 5} = \frac{6}{55}$$

(f) $\frac{-2}{9} \times \frac{33}{54}$

$$= \frac{-1}{3} \times \frac{11}{28} = \frac{-11}{84}$$

(g) $\frac{3}{13} \div \left(\frac{-4}{65}\right)$

$$= \frac{3}{13} \times \left(\frac{65}{-4}\right) = \frac{3}{1} \times \frac{-5}{4} = \frac{-15}{4}$$

(h) $\frac{-11}{9} \div \frac{12}{81}$

$$= \frac{-11}{9} \times \frac{81}{12} = \frac{-11}{1} \times \frac{9}{12}$$

$$= \frac{-99}{12} = \frac{-33}{4}$$

(i) $-4 \div \left(\frac{-3}{5}\right)$

$$= -4 \times \frac{-5}{3} = \frac{20}{3}$$

(j) $\frac{-25}{7} \div \frac{15}{14}$

$$= \frac{-25}{7} \times \frac{14}{15} = \frac{-5}{1} \times \frac{2}{3}$$

$$= \frac{-10}{3}$$

(k) $\frac{-5}{9} \times \frac{49}{56} \times \frac{-72}{35}$

$$= \frac{-5}{9} \times \frac{7}{8} \times \frac{-72}{35}$$

$$= \frac{-5}{9} \times \frac{7}{1} \times \frac{-9}{35}$$

$$= \frac{-5}{9} \times 1 \times \frac{-9}{5}$$

$$= \frac{-5}{9} \times \frac{-9}{5} = 1$$

6. Product of two rational numbers $= \frac{15}{11}$

Given number $= \frac{5}{9}$

$\therefore$ Other number $= \frac{15}{11} \div \frac{5}{9}$

$$= \frac{15}{11} \times \frac{9}{5} = \frac{3 \times 9}{11} = \frac{27}{11}$$

7. $\left[\left(\frac{-8}{5}\right) \times \frac{3}{4}\right] + \left[\frac{7}{8} \times \left(\frac{-16}{25}\right)\right]$

$$= \left[\frac{-8}{5} \times \frac{3}{4}\right] + \left[\frac{7}{8} \times \frac{-16}{25}\right]$$

$$= \left[\frac{-2 \times 3}{5}\right] + \left[\frac{7 \times -2}{25}\right]$$

$$= \left[\frac{-6}{5}\right] + \left[\frac{-14}{25}\right]$$

$$= \frac{-6}{5} - \frac{14}{25}$$

Taking L.C.M. 25

$$= \frac{-6 \times 5 - 14 \times 1}{25} = \frac{-30 - 14}{25} = \frac{-44}{25}$$

Exercise-2

1. Name the properties of addition satisfied by the following statements :

(a) $\frac{6}{17} + \frac{11}{-17} = \frac{11}{-17} + \frac{6}{17}$

Commutative for addition

(b) $\frac{5}{12} + \frac{-5}{12} = 0$

Additive inverse

(c) $\frac{6}{17} + \left(\frac{-3}{11} + \frac{2}{10}\right) = \left(\frac{6}{17} + \frac{-3}{11}\right) + \frac{2}{10}$

Associative for addition

(d) $\frac{2}{7} \times \frac{7}{2} = 1$

Multiplicative inverse

2. Solve :

(a) $\frac{2}{5} + \frac{8}{3} + \frac{4}{5} + \left(\frac{-2}{3}\right)$

$\frac{2}{5} + \frac{8}{3} + \frac{4}{5} - \frac{2}{3}$

L.C.M. of 5 and 3 is 15

$= \frac{2 \times 3 + 8 \times 5 + 4 \times 3 - 2 \times 5}{15}$

(b) $\frac{11}{12} + \frac{(-17)}{4} + \frac{-4}{9} + \frac{(-15)}{8} + \frac{13}{6}$

$= \frac{11}{12} - \frac{17}{4} - \frac{4}{9} - \frac{15}{8} + \frac{13}{6}$

L.C.M. of 12, 4, 9, 8, 6 is 72.

$$= \frac{6+40+12-10}{15} = \frac{48}{15}$$

$$= \frac{16}{5}$$

(c) $\frac{11}{12} \times \frac{-7}{8} \times \frac{-8}{15}$

$$= \frac{11}{12} \times \frac{-7 \times -1}{15}$$

$$= \frac{77}{180}$$

$$= \frac{11 \times 6 - 17 \times 18 - 4 \times 8 - 15 \times 9 + 13 \times 12}{72}$$

$$= \frac{66 - 306 - 32 - 135 + 156}{72}$$

$$= \frac{222 - 473}{72} = \frac{-251}{72}$$

(d) $\frac{5}{4} \times \frac{-3}{7} + \frac{5}{4} \times \frac{9}{13}$

$$= \frac{5 \times (-3)}{4 \times 7} + \frac{5 \times 9}{4 \times 13}$$

$$= \frac{-15}{28} + \frac{45}{52}$$

L.C.M. of 28 and 52 is 364

$$= \frac{-15 \times 13 + 45 \times 7}{364} = \frac{-195 + 315}{364}$$

$$= \frac{120}{364} = \frac{30}{91}$$

Dividing by 4

(e) $\frac{2}{9} \times \frac{7}{10} - \frac{2}{9} \times \frac{1}{5}$

$$= \frac{2 \times 7}{9 \times 10} - \frac{2 \times 1}{9 \times 5}$$

$$= \frac{14}{90} - \frac{2}{45}$$

L.C.M. of 90 and 45 is 90

$$= \frac{14 - 2 \times 2}{90} = \frac{14 - 4}{90} = \frac{10}{90} = \frac{1}{9}$$

(f) $\frac{36}{49} \div \frac{27}{-42} \times \frac{26}{-45}$

$$= \frac{36}{49} \times \frac{-42}{27} \times \frac{26}{-45}$$

$$= \frac{36}{7} \times \frac{-6}{27} \times \frac{26}{-45}$$

$$= \frac{36}{7} \times \frac{-2}{9} \times \frac{26}{-45}$$

$$= \frac{4}{7} \times (-2) \times \frac{26}{-45}$$

$$= \frac{-208}{-315} = \frac{208}{315}$$

3. Verify the property $a \times (b \times c) = (a \times b) \times c$ by taking :

(a) $a = \frac{-7}{3}, b = \frac{5}{-4}, c = \frac{9}{8}$

$$a \times (b \times c) = (a \times b) \times c$$

Putting value a, b and c in the given condition,

$$\frac{-7}{3} \times \left(\frac{5}{-4} \times \frac{9}{8}\right) = \left(\frac{-7}{3} \times \frac{5}{-4}\right) \times \frac{9}{8}$$

L.H.S. $= \frac{-7}{3} \times \left(\frac{5}{-4} \times \frac{9}{8}\right)$

$$= \frac{-7}{3} \times \left(\frac{5 \times 9}{-4 \times 8}\right)$$

$$= \frac{-7}{3} \times \frac{45}{-32} = \frac{-7 \times 15}{-32}$$
$$= \frac{105}{32}$$

Now R.H.S. $= \left(\frac{-7}{3} \times \frac{5}{-4}\right) \times \frac{9}{8}$

$$= \frac{-7 \times 5}{3 \times (-4)} \times \frac{9}{8} = \frac{-35}{-12} \times \frac{9}{8}$$
$$= \frac{35 \times 3}{4 \times 8} = \frac{105}{32}$$

∴ R.H.S. = L.H.S.

(b) $a = \frac{-3}{7}, b = \frac{4}{5}, c = \frac{-5}{8}$

Putting value a, b and c into the given condition,

$$a \times (b \times c) = (a \times b) \times c$$
$$= \frac{-3}{7} \times \left(\frac{4}{5} \times \frac{-5}{8}\right) = \left(\frac{-3}{7} \times \frac{4}{5}\right) \times \frac{-5}{8}$$

L.H.S. $= \frac{-3}{7} \times \left(\frac{4}{5} \times \frac{-5}{8}\right)$

$$= \frac{-3}{7} \times \left(\frac{-4}{8}\right) = \frac{-3}{7} \times \frac{-4}{8}$$
$$= \frac{-3 \times (-4)}{7 \times 8}$$
$$= \frac{-3 \times (-2)}{7 \times 4} = \frac{6}{28}$$
$$= \frac{3}{14}$$

.

Now R.H.S. $= \left(\frac{-3}{7} \times \frac{4}{5}\right) \times \frac{-5}{8}$

$$= \left(\frac{-3 \times 4}{7 \times 5}\right) \times \frac{-5}{8}$$
$$= \frac{-12}{35} \times \frac{-5}{8} = \frac{-3}{35} \times \frac{-5}{2}$$
$$= \frac{-3}{7} \times \frac{-1}{2}$$
$$= \frac{-3 \times (-1)}{7 \times 2} = \frac{3}{14}$$

∴ R.H.S. = L.H.S.

4. Verify the following and name the property :

(a) $\frac{-7}{6} + \left(\frac{3}{8} + \frac{1}{4}\right) = \left(\frac{-7}{6} + \frac{3}{8}\right) + \frac{1}{4}$ (b) $\frac{4}{7} \times \left(\frac{2}{3} + \frac{3}{4}\right) = \frac{4}{7} \times \frac{2}{3} + \frac{4}{7} \times \frac{3}{4}$

$$\text{L.H.S.} = \frac{-7}{6} + \left(\frac{3+1\times 2}{8}\right)$$

$$= \frac{-7}{6} + \frac{5}{8} = \frac{-7\times 4 + 5\times 3}{24}$$

$$= \frac{-28+15}{24}$$

$$= \frac{-13}{24}$$

$$\text{R.H.S.} = \left(\frac{-7}{6} + \frac{3}{8}\right) + \frac{1}{4}$$

$$= \left(\frac{-7\times 4 + 3\times 3}{24}\right) + \frac{1}{4}$$

$$= \left(\frac{-28+9}{24}\right) + \frac{1}{4} = \frac{-19}{24} + \frac{1}{4}$$

$$= \frac{-19+6}{24} = \frac{-13}{24}$$

∴ L.H.S. = R.H.S.

This is an associative property.

(c) $\frac{14}{9} \times \left(\frac{-3}{7}\right) = \frac{-3}{7} \times \frac{14}{9}$

$$\text{L.H.S.} = \frac{14}{3} \times \frac{-1}{7}$$

$$= \frac{2}{3} \times \frac{-1}{1} = \frac{-2}{3}$$

$$\text{R.H.S.} = \frac{-3}{7} \times \frac{14}{9}$$

$$= \frac{-1}{7} \times \frac{14}{3} = \frac{-1}{1} \times \frac{2}{3}$$

$$= \frac{-2}{3}$$

∴ L.H.S. = R.H.S.

So, this is a multiplication property.

$$\text{L.H.S.} = \frac{4}{7} \times \left(\frac{2}{3} + \frac{3}{4}\right)$$

$$= \frac{4}{7} \times \left(\frac{2\times 4 + 3\times 3}{12}\right)$$

$$= \frac{4}{7} \times \left(\frac{8+9}{12}\right) = \frac{4}{7} \times \left(\frac{17}{12}\right)$$

$$= \frac{4}{7} \times \frac{17}{12}$$

$$= \frac{1}{7} \times \frac{17}{3} = \frac{17}{21}$$

$$\text{R.H.S.} = \frac{4}{7} \times \frac{2}{3} + \frac{4}{7} \times \frac{3}{4}$$

$$= \frac{8}{21} + \frac{12}{28}$$

$$= \frac{8}{21} + \frac{3}{7} = \frac{8 + 3\times 3}{21}$$

$$= \frac{8+9}{21} = \frac{17}{21}$$

∴ L.H.S. = R.H.S.

This is a distributive property.

(d) $\frac{3}{7} \times \left(\frac{7}{8} - \frac{3}{4}\right) = \frac{3}{7} \times \frac{7}{8} - \frac{3}{7} \times \frac{3}{4}$

$$\text{L.H.S.} = \frac{3}{7} \times \left(\frac{7}{8} - \frac{3}{4}\right)$$

$$= \frac{3}{7} \times \left(\frac{7\times 2 - 3\times 4}{16}\right)$$

$$= \frac{3}{7} \times \left(\frac{14-12}{16}\right) = \frac{3}{7} \times \left(\frac{2}{16}\right)$$

$$= \frac{3}{7} \times \frac{1}{8} \quad = \frac{3}{56}$$

$$\text{R.H.S.} = \frac{3}{7} \times \frac{7}{8} - \frac{3}{7} \times \frac{3}{4}$$

$$= \frac{3}{1} \times \frac{1}{8} - \frac{9}{28}$$

$$= \frac{3}{8} - \frac{9}{28} = \frac{3\times 7 - 9\times 2}{56}$$

$$= \frac{21-18}{56} = \frac{3}{56}$$

∴ L.H.S. = R.H.S.

5. Write :

(a) The rational number which is its own additive inverse.

(b) The rational number which is its own multiplicative inverse.

(c) The rational number that does not have a reciprocal.

(a) 0 (b) 1 (c) 0

6. (a) The additive inverse of $\frac{-15}{7}$

$$\frac{-15}{7}+\left(\frac{15}{7}\right)=\frac{-15}{7}+\frac{15}{7}=0$$

So, $\frac{15}{7}$ is additive inverse of $\frac{-15}{7}$.

(b) The multiplicate inverse of $\frac{-4}{7}$

$$=\frac{-4}{7}\times\frac{7}{-4}=1$$

So, $\frac{7}{-4}$ is multiplicate inverse of $\frac{-4}{7}$.

7. Divide the sum of $\frac{2}{9}$ and $\frac{4}{7}$ by their difference.

Given condition $=\dfrac{\frac{2}{9}+\frac{4}{7}}{\frac{2}{9}-\frac{4}{7}}$

$$=\dfrac{\frac{2\times 7+4\times 9}{9\times 7}}{\frac{2\times 7-4\times 9}{9\times 7}}$$

$$=\dfrac{\frac{(14+36)}{63}}{\frac{(14-36)}{63}}=\dfrac{\frac{50}{63}}{\frac{-22}{63}}$$

$$=\frac{50}{63}\times\frac{63}{-22}=\frac{50}{-22}$$

$$=\frac{-25}{11}$$

8. The product of two rational numbers is $\frac{-8}{9}$. If one of them is $\frac{10}{3}$, find the other.

Solution : Let other number be x

As per question, $\frac{10}{3}\times x=\frac{-8}{9}$

$$\frac{10x}{3}=\frac{-8}{9}$$

$$10x=\frac{-8}{9}\times 3$$

$$10x=\frac{-8}{3}\quad\text{or } x=\frac{-8}{3\times 10}$$

$$x=\frac{-4}{3\times 5}$$

$\therefore \qquad x = \frac{-4}{15}$

9. A container has a capacity of $84\frac{4}{5}$ litres. If the container is filled with $11\frac{2}{3}$ litres of oil, find how much oil the container can hold.

Ans. Given

Capacity of a container $= 84\frac{4}{5}$ litres

If container is filled with $= 11\frac{2}{3}$ litres oil

How much oil the container can hold = ?

The container can hold = (total capacity of a container – oil already filled)

$$= \left(84\frac{4}{5} - 11\frac{2}{3}\right)$$

$$= \left(\frac{424}{5} - \frac{35}{3}\right) = \frac{(424 \times 3 - 35 \times 5)}{15}$$

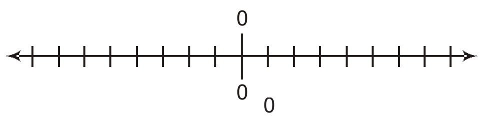
$$= \frac{(1272 - 175)}{15} = \frac{1097}{15}$$

$$= 73\frac{2}{15} \text{ litres}$$

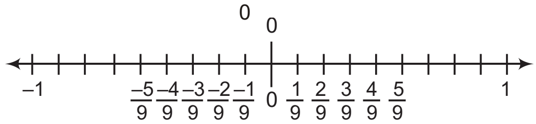
Exercise-3

1. Represent the following on the number line :

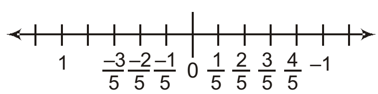
(a) $\frac{8}{-5} = \frac{-8}{5}$



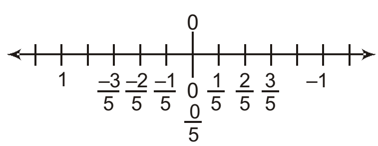
(b) $\frac{-5}{9}$



(c) $\frac{-3}{5}$



(d) $\frac{0}{5}$



2. Which of the following rational numbers are to the right of zero and which of them are to the left of zero?

Solution : $\frac{1}{10}$ and $\frac{-12}{-13}$ *i.e.,* $\frac{12}{13}$ are to the right of zero, while $\frac{-15}{19}$ and $\frac{-4}{7}$ are to the left of zero.

3. Which rational number is represented by the point A in the number line given below :

Solution : $A = -2 \cdot 6$

4. Write five rational numbers between $\frac{-1}{3}$ and $\frac{1}{3}$.

Solution : Evidently 0 is a rational number between $\frac{-1}{3}$ and $\frac{1}{3}$,

Now rational number between $\frac{-1}{3}$ and $0 = \frac{\left(\frac{-1}{3}\right)+0}{2} = -\frac{1}{6}$

Rational No. between $\frac{-1}{6}$ and $0 = \frac{\left(\frac{-1}{6}\right)+0}{2} = -\frac{1}{12}$

Again, rational No. between 0 and $\frac{1}{3} = \frac{0+\left(\frac{1}{3}\right)}{2} = \frac{1}{6}$

Rational No. between 0 and $\frac{1}{6} = \frac{0+\left(\frac{1}{6}\right)}{2} = \frac{1}{12}$

$\therefore$ Required number are $-\frac{1}{6}, -\frac{1}{12}, 0, \frac{1}{6}, \frac{1}{12}$.

5. Write any four rational number between 2 and 3.

Solution : A rational number between 2 and $3 = \frac{2+3}{2} = \frac{5}{2}$

$\therefore \quad 2 < \frac{5}{2} < 3$

Now we can find a rational number between $\left(2 \text{ and } \frac{5}{2}\right)$ and between $\left(\frac{5}{2} \text{ and } 3\right)$

$\therefore$ Rational number between $\left(2 \text{ and } \frac{5}{2}\right) = \frac{2+(5/2)}{2}$

$$= \frac{\frac{4+5}{2}}{2} = \frac{9}{4}$$

Rational number between $\left(\frac{5}{2} \text{ and } 3\right) = \frac{\left(\frac{5}{2}\right)+3}{2}$

$$= \frac{\frac{5+6}{2}}{2} = \frac{11}{4}$$

Now, $2 < \frac{9}{4} < \frac{5}{2} < \frac{11}{4} < 3$

$\therefore$ Rational number between 2 and $\frac{9}{4} = \frac{2+\left(\frac{9}{4}\right)}{2}$

$$= \frac{\frac{8+9}{4}}{2} = \frac{17}{8}$$

$\therefore$ The required four rational numbers are $\frac{9}{4}, \frac{17}{8}, \frac{5}{2}, \frac{11}{4}$ **Ans.**

6. Write any 10 rational number between 0 and 2.

Solution : A rational number between 0 and 2 = $\frac{0+2}{2} = 1$

Now, $0 < 1 < 2$

A rational number between (0 and 1) = $\frac{0+1}{2} = \frac{1}{2}$

A rational number between (1 and 2) = $\frac{1+2}{2} = \frac{3}{2}$

Now, $0 < \frac{1}{2} < 1 \frac{3}{2} < 2$

Here we can write $\frac{1}{2} = \frac{10}{20}$ and $\frac{3}{2} = \frac{30}{20}$

So, we have $\frac{11}{20}, \frac{12}{20}, \frac{13}{20}, \frac{14}{20}, \frac{15}{20}, \ldots, \frac{31}{20}, \frac{32}{20}, \ldots, \frac{38}{20}, \frac{39}{20}, \frac{40}{20} = 2$

7. Find six rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$.

Solution : A rational number between $\frac{2}{3}$ and $\frac{4}{5} = \frac{\frac{2}{3}+\frac{4}{5}}{2}$

$$= \frac{\frac{10+12}{15}}{2} = \frac{22}{30}$$

Now, $\frac{2}{3} < \frac{22}{30} < \frac{4}{5}$

Here $\frac{2}{3} = \frac{20}{30}$ and $\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$

$\therefore$ $\frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$

Inserting rational numbers with denominator 60 between every pair with denominator 30, we get

$\therefore$ $\frac{20}{30} < \frac{41}{60} < \frac{21}{30} < \frac{43}{60} < \frac{22}{30} < \frac{45}{60} < \frac{23}{30} < \frac{47}{60} < \frac{24}{30}$

Hence the equired rational numbes are :

$$\frac{41}{60}, \frac{21}{30}, \frac{43}{60}, \frac{22}{30}, \frac{45}{60}, \frac{23}{30} \text{ and } \frac{47}{60}$$

Chapter-2 : Exponents and Powers

Exercise-1

1. Evaluate :

(a) $\left(\frac{1}{3}\right)^{-4}$ (b) $(-5)^4$

$= \frac{1}{\left(\frac{1}{3}\right)^4} = \left(\frac{3}{1}\right)^4 = \frac{3\times3\times3\times3}{1\times1\times1\times1}$

$= 81$

$= (-5)\times(-5)\times(-5)\times(-5)$

$= 625$

(c) $\left(\frac{2}{5}\right)^{-3}$

$= \frac{1}{\left(\frac{2}{5}\right)^3} = \left(\frac{5}{2}\right)^3$

$= \frac{5}{2}\times\frac{5}{2}\times\frac{5}{2}$

$= \frac{125}{8}$

(d) $\left[\left(\frac{12}{25}\right)^{-1}\right]^2$

$= \left(\frac{12}{25}\right)^{-2} = \left(\frac{25}{12}\right)^2$

$= \frac{25}{12}\times\frac{25}{12} = \frac{625}{144}$

(e) $6^{-1}\times6^3$

$= \frac{1}{6}\times6\times6\times6$

$= 6\times6 = 36$

2. Simplify :

(a) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^2$

$= \frac{1}{\left(\frac{1}{2}\right)^2} + \frac{1}{\left(\frac{1}{3}\right)^2} + \left(\frac{1}{4}\right)^2$

$= \left(\frac{2}{1}\right)^2 + \left(\frac{3}{1}\right)^2 + \left(\frac{1}{4}\right)^2$

$= \frac{2\times2}{1\times1} + \frac{3\times3}{1\times1} + \frac{1}{4}\times\frac{1}{4}$

$= \frac{4}{1} + \frac{9}{1} + \frac{1}{16}$

Taking :L.C.M.

$= \frac{4\times16+9\times16+1}{16}$

$= \frac{64+144+1}{16} = \frac{209}{16}$

(b) $\left[\left(\frac{3}{7}\right)^{-2}\right]^4$

$= \left[\left(\frac{3}{7}\right)^{-2\times4}\right] = \left(\frac{3}{7}\right)^{-8}$

$= \left(\frac{7}{3}\right)^8$

$= \frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}$

$= \frac{5764801}{6561}$

(c) $(5^2 - 3^2) - \left(\frac{5}{4}\right)^2$

(d) $\left[\left(\frac{1}{3}\right)^{-1}\times(-9)^{-1}\right]^{-1}$

$= (25-9)\ -\frac{5}{4}\times\frac{5}{4}$

$=16-\frac{25}{16}=\frac{16\times16-25}{16}=\frac{256-25}{16}$

$=\frac{231}{16}$

$=\left[(3)^1\times\left(\frac{1}{-9}\right)^1\right]^{-1}$

$=\left[\left(3\times\frac{1}{-9}\right)\right]^{-1}=\left(\frac{1}{-3}\right)^{-1}$

$=-3$

(e) $\left(\frac{-5}{8}\right)^4\times\left(\frac{-5}{8}\right)^7\div\left(\frac{-5}{8}\right)^3$

$=\left(\frac{-5}{8}\right)^4\times\left(\frac{-5}{8}\right)^7\times\left(\frac{8}{-5}\right)^3$

$=\left(\frac{-5}{8}\right)^4\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{8}{-5}\times\frac{8}{-5}\times\frac{8}{-5}$

$=\left(\frac{-5}{8}\right)^4\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}=\left(\frac{-5}{8}\right)^8$

$=\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}\times\frac{-5}{8}$

$=\frac{390625}{66777216}$

(f) $(4^{-1}\div6^{-1})^2$

$=\left(\frac{1}{4}\div\frac{1}{6}\right)^2=\left(\frac{1}{4}\times6\right)^2$

$=\left(\frac{3}{2}\right)^2$

$=\frac{3}{2}\times\frac{3}{2}=\frac{9}{4}$

3. Evaluate :

(a) $(5^0+4^{-1})\times2^2$

$=\left(5^0+\frac{1}{4}\right)\times2^2$

$=\left(1+\frac{1}{4}\right)\times2\times2=\left(\frac{1\times4+1}{4}\right)\times4$

$=\left(\frac{4+1}{4}\right)\times4=\frac{5}{4}\times4$

$=5$

(b) $(6^{-2}\times2^{-2})^{-2}$

$=\left[\left(\frac{1}{6}\right)^2\times\left(\frac{1}{2}\right)^2\right]^{-2}$

$=\left(\frac{1}{36}\times\frac{1}{4}\right)^{-2}=\left(\frac{1\times1}{36\times4}\right)^{-2}$

$=\left(\frac{1}{144}\right)^{-2}=(144)^2$

$=20736$

(c) $16^{\frac{1}{2}}(16^{\frac{1}{2}}-2)$

$= 4^{2\times\frac{1}{2}}\left(4^{2\times\frac{1}{2}}-2\right) = 4\,(4-2)$

$= 4\times 2 = 8$

(d) $\dfrac{(2^3\times 3^4)\times(-5)^3}{60\times(-2)^5}$

$= \dfrac{2^3\times 3^4\times(-5)^3}{2\times 2\times 3\times 5\times(-2)^5}$

$= \dfrac{2\times 3^3\times(-5)^3}{5\times(-2)^5} = \dfrac{-2\times 27\times 125}{-5\times 32}$

$= \dfrac{27\times 25}{16} = \dfrac{675}{16}$

(e) $[4^{-1}-3^{-3}+6^{-2}]^{-1}$

$= \left[\dfrac{1}{4}-\dfrac{1}{3^3}+\dfrac{1}{6^2}\right]^{-1}$

$= \left[\dfrac{1}{4}-\dfrac{1}{27}+\dfrac{1}{36}\right]^{-1} = \left[\dfrac{1\times 27-1\times 4+1\times 3}{108}\right]^{-1}$

$= \left[\dfrac{27-4+3}{108}\right]^{-1} = \left[\dfrac{26}{108}\right]^{-1}$

$= \left[\dfrac{13}{54}\right]^{-1} = \dfrac{54}{13}$

(f) $\left[\left(\dfrac{-8}{13}\right)^{-1}+\left(\dfrac{14}{5}\right)^{-1}\right]^2$

$= \left[\dfrac{-13}{8}+\dfrac{5}{14}\right]^2$

$= \left[\dfrac{-13\times 7+5\times 4}{56}\right]^2 = \left[\dfrac{-91+20}{56}\right]^2$

$= \left(\dfrac{-71}{56}\right)^2 = \dfrac{-71\times(-71)}{56\times 56}$

$= \dfrac{5041}{3136}$

4. Find the value of x :

(a) $\left(\dfrac{2}{3}\right)^{-4}\times\left(\dfrac{2}{3}\right)^{-8} = \left(\dfrac{2}{3}\right)^{4x}$

$\left(\dfrac{2}{3}\right)^{-4-8} = \left(\dfrac{2}{3}\right)^{4x}$

$\left(\dfrac{2}{3}\right)^{-12} = \left(\dfrac{2}{3}\right)^{4x}$

On equating the exponents since bases are equal, we have

$-12 = 4x$

$4x = -12$

$x = \dfrac{-12}{4}$

$x = -3$

(b) $3^{x-2}\div 3^{-3} = 3^4$

or $3^{(x-2)}\times\dfrac{1}{3^{-3}} = 3^4$

or $3^{(x-2)}\times 3^3 = 3^4$

or $3^{(x-2)+3} = 3^4$

or $$3^{x+1} = 3^4$$

On equating the exponents since bases are equal, we have

$$x + 1 = 4$$
$$x = 4 - 1$$
$$x = 3$$

(c) $\left(\frac{1}{3}\right)^{2x} \times \left(\frac{1}{3}\right)^2 = 3^4$

$$\left(\frac{1}{3}\right)^{2x} \times \left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^{-4}$$
$$\left(\frac{1}{3}\right)^{2x+2} = \left(\frac{1}{3}\right)^{-4}$$

On equating the exponents since bases are equal, we have

$$2x + 2 = -4$$
$$2x = -4 - 2$$
$$2x = -6$$
$$x = \frac{-6}{2}$$
$$x = -3$$

(d) $2^{3x-5} = \frac{1}{4}$

$$2^{3x-5} = \left(\frac{1}{2}\right)^2$$
$$2^{3x-5} = 2^{-2}$$

On equating the exponents since bases are equal, we have

$$3x - 5 = -2$$
$$3x = -2 + 5$$
$$3x = 3$$
$$x = \frac{3}{3}$$
$$x = 1$$

5. Verify that $(x \times y)^m = x^m \times y^m$ for $x = \frac{2}{3}$, $y = \frac{5}{7}$ and $m = 2$

$$(x \times y)^m = x^m \times y^m$$

Given $x = \frac{2}{3}$, $y = \frac{5}{7}$, $m = 2$

Taking L.H.S. $= (x \times y)^m$

Putting the values of x, y and m

$$= \left(\frac{2}{3} \times \frac{5}{7}\right)^2$$

$$= \left(\frac{10}{21}\right)^2 = \frac{100}{441}$$

R.H.S. = $x^m \times y^m$

Putting the values of x, y and m

$$= \left(\frac{2}{3}\right)^2 \times \left(\frac{5}{7}\right)^2$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{5}{7} \times \frac{5}{7}$$

$$= \frac{4}{9} \times \frac{25}{49}$$

$$= \frac{100}{441}$$

∴ L.H.S. = R.H.S.

6. Let the number be = x

As per question, $$\left(\frac{1}{2}\right)^{-1} \times x = \left(\frac{-5}{9}\right)^{-1}$$

or $$2 \times x = \left(\frac{9}{-5}\right)$$

$$x = \frac{9}{-5 \times 2}$$

∴ $$x = \frac{-9}{10}$$

The number by which $\left(\frac{1}{2}\right)^{-1}$ should be multiplied so that product becomes $\left(\frac{-5}{9}\right)^{-1}$ is $\frac{-9}{10}$.

7. Let the number be = x

As per question, $$(-40)^{-1} \div x = (5)^{-1}$$

or $$\left(-\frac{1}{40}\right) \div x = \frac{1}{5}$$

or $$\left(-\frac{1}{40}\right) \times \frac{1}{x} = \frac{1}{5}$$

or $$-\frac{1}{40x} = \frac{1}{5}$$

or $$40x = -5$$

or $$x = -\frac{5}{40}$$

∴ $$x = \frac{-1}{8}$$

The number by which $(-40)^{-1}$ should be divided so that the quotient becomes $(5)^{-1}$ is $\frac{-1}{8}$.

8. Find n so that $2^{11} \div 2^{5} = 2^{-3} \times 2^{2n-1}$.

$$2^{11} \div 2^{5} = 2^{-3} \times 2^{2n-1}$$

$$2^{11} \div \frac{1}{2^{-5}} = 2^{-3} \times 2^{2n-1}$$

$$2^{11} \times 2^{-5} = 2^{-3} \times 2^{2n-1}$$

$$2^{11-5} = 2^{2n-1-3}$$

$$2^{6} = 2^{2n-4}$$

On equating the exponents since bases are equal, we have

$$6 = 2n - 4$$

or $\quad 2n = 6 + 4$

or $\quad 2n = 10$

$\therefore \quad n = \frac{10}{2}$

$$n = 5$$

9. Express 4^{-3} with base 2.

$$4^{-3} = (2^{2})^{-3}$$

$$= 2^{2 \times -3}$$

$$= 2^{-6}$$

10. Simplify :

(a) $\dfrac{5^{2} \times P^{-4}}{5^{3} \times 10 \times P^{-8}}$

$$= \frac{5^{2} \times P^{-4}}{5^{3} \times 2 \times 5 \times P^{-8}}$$

$$= \frac{5^{2} \times P^{-4}}{5^{4} \times 2 \times P^{-8}}$$

$$= \frac{P^{-4+8}}{5^{4-2} \times 2}$$

$$= \frac{P^{4}}{5^{2} \times 2}$$

$$= \frac{P^{4}}{50}$$

(b) $\left[(2^{0} + 3^{-1}) \times 9^{2}\right]$

$$= \left[\left(1 + \frac{1}{3}\right) \times 9^{2}\right]$$

$$= \left[\left(\frac{3+1}{3}\right) \times 9^{2}\right]$$

$$= \left[\frac{4}{3} \times 81\right]$$

$$= 4 \times 27$$

$$= 108$$

(c) $\left[\left(\dfrac{2}{3}\right)^{-1} - \left(\dfrac{2}{5}\right)^{-1}\right]^{-2} \div \left(\dfrac{3}{4}\right)^{-3}$

$$= \left[\frac{3}{2} - \frac{5}{2}\right]^{-2} \div \left(\frac{3}{4}\right)^{-3}$$

$$= \left[\frac{3-5}{2}\right]^{-2} \div \left(\frac{3}{4}\right)^{-3}$$

(d) $(6^{0} - 2^{0}) \times (6^{0} + 2^{0})$

$$= (1-1) \times (1+1)$$

$$= 0 \times 2$$

$$= \left[\frac{-2}{2}\right]^{-2} \div \left(\frac{3}{4}\right)^{-3}$$

$$= 0$$

$$= (-1)^{-2} \div \left(\frac{4}{3}\right)^{3}$$

$$= 1 \times \left(\frac{3}{4}\right)^{3}$$

$$= \frac{27}{64}$$

(e) $\dfrac{25^{\frac{1}{2}} + 36^{\frac{3}{2}} - 4^{\frac{3}{2}}}{125^{\frac{2}{3}}}$

$$= \frac{(5^2)^{\frac{1}{2}} + (6^2)^{\frac{3}{2}} - (2^2)^{\frac{3}{2}}}{(5^3)^{\frac{2}{3}}}$$

$$= \frac{5^{2\times\frac{1}{2}} + 6^{2\times\frac{3}{2}} - 2^{2\times\frac{3}{2}}}{5^{3\times\frac{2}{3}}}$$

$$= \frac{5 + 6^3 - 2^3}{5^2}$$

$$= \frac{5 + 216 - 8}{25}$$

$$= \frac{221 - 8}{25} = \frac{213}{25}$$

Exercise-2

1. Write the following in standard form :

(a) $625003298 \cdot 25$

$$= \frac{62500329825}{100}$$

$$= 62500329825 \times 10^{-2}$$

$$= 6 \cdot 2500329825 \times 10^{-2} \times 10^{10}$$

$$= 6 \cdot 2500329825 \times 10^{8}$$

(b) $100 \cdot 001 \times 10^3$

$$= \frac{100001}{1000} \times 10^3$$

$$= 100001 \times 10^3 \times 10^{-3}$$

$$= 100001$$

$$= 1 \cdot 00001 \times 10^5$$

(c) 418700000

$$= 4 \cdot 187 \times 10^8$$

(d) $0 \cdot 0000000029$

$$= \frac{29}{10^{10}}$$

$$= 29 \times 10^{-10} \text{ or } 2 \cdot 9 \times 10^{-9}$$

(e) 343600

$$= 3 \cdot 436 \times 10^5$$

2. Write the following in usual form :

(a) 3×10^{-8}

$=\dfrac{3}{100000000}$

$=0\cdot00000003$

(b) $724\cdot9\times10^{3}$

$=724900$

(c) $5\cdot3\times10^{-13}$

$=\dfrac{5\cdot3}{10000000000000}$

$=0\cdot00000000000053$

(d) $9\cdot7137\times10^{13}$

$=97137000000000$

(e) $0\cdot001\times10^{6}$

$=1000$

3. Express the following numbers in standard form :

(a) 300,000,000

$=3\times10^{8}$ m/s

(b) 384,460,000

$=3\cdot8446\times10^{8}$

(c) $0\cdot00000001275$ km

$=\dfrac{1275}{10^{11}}$

$=1275\times10^{-11}$

$=1\cdot275\times10^{3}\times10^{-11}$ $=1\cdot275\times10^{-8}$ km

4. Consider the diameter of the sun i.e. $1\cdot4\times10^{9}$ and the earth i.e. $1\cdot2756\times10^{7}$. Now how to compare these two?

Solution : Ratio of diametes of the sun and the earth

$$=\frac{\text{Dia. of the sun}}{\text{Dia. of the earth}}$$

$$=\frac{1\cdot4\times10^{9}}{1\cdot2756\times10^{7}}$$

$$=\frac{1\cdot4\times10^{2}}{1\cdot2756}=\frac{140}{1\cdot2756}$$

$$=\frac{140\times10^{4}}{12756}$$

$$=\frac{1400000}{12756}=\frac{109\cdot75}{1}$$

$$=\frac{110}{1}=110:1$$

5. The mass of a cell is $=1500\times10^{-20}$ g

$\therefore$ The mass of 100 cells $=1500\times10^{-20}\times100$ g

$$=1500\times10^{-18}$$

$$=\frac{1500}{10^{18}}$$

$$=\frac{15\times10^{2}}{10^{18}}=\frac{15}{10^{16}}=1\cdot5\times10^{-15}$$

Chapter-3 : Squares and Square Roots

Exercise-1

1. Find the square of the following :

(a) 14

$(14)^2 = 14 \times 14$

$= 196$

(b) 23

$(23)^2 = 23 \times 23$

$= 529$

(c) 62

$(62)^2 = 62 \times 62$

$= 3844$

(d) $\frac{19}{20}$

$\left(\frac{19}{20}\right)^2 = \frac{19}{20} \times \frac{19}{20}$

$= \frac{361}{400}$

2. What will be the ones digit in the squares of the following numbers?

(a) 860

$860 = (0)^2$

$= 0$

(b) 565

$565 = (5)^2 = 25$

$= 5$

(c) 81

$81 = (1)^2$

$= 1 \times 1 = 1$

(d) $789 = (9)^2$

$= 81 = 1$

3. Find the Pythagorean triplets of the numbers, the smallest of which is given below :

(a) 20

Given that

$2m = 20$

or $m = 10$

So, $m^2 - 1 = (10)^2 - 1$

$= 100 - 1 = 99$

$m^2 + 1 = (10)^2 + 1$

$= 100 + 1$

$= 101$

Therefore, (20, 99, 101) is the Pythagorean triplet.

(b) 12

Given that

$2m = 12$

or $m = 6$

So, $m^2 - 1 = (6)^2 - 1$

$= 36 - 1$

$= 35$

$m^2 + 1 = (6)^2 + 1$

$= 36 + 1 = 37$

Therefore, (12, 35, 37) is the Pythagorean triplet.

(c) 18

Given that $2m = 18$

$m = 9$

So, $m^2 - 1 = (9)^2 - 1$

$= 81 - 1$

$= 80$

$m^2 + 1 = (9)^2 + 1$

$= 81 + 1$

$= 82$

(d) 8

Given that $2m = 8$

$m = 4$

So, $m^2 - 1 = (4)^2 - 1$

$= 16 - 1$

$= 15$

$m^2 + 1 = (4)^2 + 1$

$= 16 + 1 = 17$

Therefore, (8, 15, 17) is the Pythagorean triplet

Therefore, (18, 80, 82) is the Pythagorean triplet.

4. Find the smallest number by which the following have to be multiplied to make them a perfect square :

(a) 2028

First we find the factors of 2028

$$2028 = 2 \times 2 \times 3 \times 13 \times 13$$

2	2028
2	1014
3	507
13	169
13	13
	1

Making pairs of the factors of 2028, we find that 3 does not have a pair. If 2028 is multiplied by 3, then all the factors of 2028 will become pairs and the number so obtained will be a perfect square.

Therefore, the smallest number by which 2028 should be multiplied to make it a perfect square is 3

(b) 1792

First we find the factors of 1792

$$1792 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7$$

2	1792
2	896
2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

Making pairs of the factors of 1792, we find that 7 does not have a pair. If 1792 is multiplied by 7, then all the factors of 1792 will become pairs and the number so obtained will be perfect square. Therefore, the smallest number by which 1792 should be multiplied to make it a perfect square is 7.

(c) 2904

First we find the factors of 2904

$$2904 = 2 \times 2 \times (2 \times 3) \times 11 \times 11$$

2	2904
2	1452
2	726
3	363
11	121
11	11
	1

Making pairs of the factors of 2904, we find that 6 does not have a pair. If 2904 is multiplied by 6, then all the factors of 2904 will become pairs and the number by which 2904 should be multiplied to make it a perfect square is 6.

5. Using the properties, find the following :

(a) $(16)^2 - (15)^2$

$$= (16-15)\,(16+15)$$
$$= 1 \times (16+15)$$
$$= 31$$

(b) $(879)^2 - (878)^2$

$$= (879-878)\,(879+878)$$
$$= 1 \times (879+878)$$
$$= 1757$$

(c) $(95)^2$ $= (90+5)\,(90+5)$

$$= 90\,(90+5) + 5\,(90+5)$$
$$= 90 \times 90 + 90 \times 5 + 5 \times 90 + 5 \times 5$$
$$= 8100 + 450 + 450 + 25 = 9025$$

(d) $1+3+5+7+9+11$

We know that

$1+3+5+7+9+11 = n^2$

Here $n = 6$

$\therefore$ Sum $= 6^2 = 36$

(e) $1+3+5+7+9+11+13+15+17+19+21$

Here $n = 11$,

$\therefore$ Sum $= n^2 = (11)^2 = 121$

6. Observe the following pattern and fill in the blanks :

$$2^2+3^2+6^2=7^2$$
$$3^2+4^2+12^2=13^2$$
$$4^2+5^2+20^2=21^2$$
$$5^2+6^2+30^2=31^2$$
$$6^2+\mathbf{7^2}+\mathbf{42^2}=43^2$$
$$\mathbf{8^2}+\mathbf{9^2}+\mathbf{72^2}=73^2$$

7. How many natural numbers lie between 8^2 and $(9)^2$?
There are 16 Natural numbers between $8^2=64$ and $9^2=81$

Exercise-2

1. Find the square root of the following by prime factorisation :

(a) 11664

$11664=4\times4\times3\times3\times9\times9$

$\therefore\quad \sqrt{11664}=4\times3\times9$

$=108$

4	11664
4	2916
3	729
3	243
9	81
9	9
	1

(b) 576

$576=3\times3\times8\times8$

$\therefore\quad \sqrt{576}=3\times8=24$

3	576
3	192
8	64
8	8
	1

(c) 1764

$1764=2\times2\times3\times3\times7\times7$

$\therefore\quad \sqrt{1764}=2\times3\times7$

$=42$

4	1764
3	441
3	147
7	49
7	7
	1

(d) 22201

$22201=149\times149$

$\therefore\quad \sqrt{22201}=149$

149	22201
149	149
	1

2. Find the square root of the following by long division method :

(a) 5776

The square root of the given number 5776

$\sqrt{5776}=76$

	76
7	$\overline{57}\,\overline{76}$
	49
146	876
	876
	0

(b) 2304

The square root of the given number 2304.

$\sqrt{2304} = 48$

	48
4	$\overline{23}\,\overline{04}$
	16
88	704
	704
	0

(c) 12544

The square root of the given number 12544.

$\sqrt{12544} = 112$

	112
1	$\overline{1}\,\overline{25}\,\overline{44}$
	1
21	25
	21
222	444
	444
	0

(d) 390625

The square root of the given number 390625.

$\sqrt{390625} = 625$

	625
6	$\overline{39}\,\overline{06}\,\overline{25}$
	36
122	306
	244
1245	6225
	6225
	0

3. Find the square root of the following by repeated subtracted method :

(a) 36

$36 - 1 = 35$ $\quad$ $35 - 3 = 32$

$32 - 5 = 27$ $\quad$ $27 - 7 = 20$

$20 - 9 = 11$ $\quad$ $11 - 11 = 0$

Now count the number of steps. Since 0 is obtained in six steps, the square root of 36 is 6, *i.e.* $\sqrt{36} = 6$

(b) 121

$121 - 1 = 120$ $\quad$ $120 - 3 = 117$

$117 - 5 = 112$ $\quad$ $112 - 7 = 105$

$105 - 9 = 96$ $\quad$ $96 - 11 = 85$

$85 - 13 = 72$ $\quad$ $72 - 15 = 57$

$57 - 17 = 40$ $\quad$ $40 - 19 = 21$

$21 - 21 = 0$

Now count the steps. Since 0 is obtained in '11' steps, square root of 121 is '11' *i.e.* $\sqrt{121} = 11$

(c) 49

$49 - 1 = 48$ $\quad$ $48 - 3 = 45$

$45 - 5 = 40$ $\quad$ $40 - 7 = 33$

$33 - 9 = 24$ $\quad$ $24 - 11 = 13$

$13 - 13 = 0$

Now count the steps. Since 0 is obtained in '7' steps , square root of 49 is '7' *i.e.* $\sqrt{49} = 7$

(d) 64

$64 - 1 = 63$ $\qquad 63 - 3 = 60$

$60 - 5 = 55$ $\qquad 55 - 7 = 48$

$48 - 9 = 39$ $\qquad 39 - 11 = 28$

$28 - 13 = 15$ $\qquad 15 - 15 = 0$

Now count the steps. Since 0 is obtaied in '8' steps, square root of 64 is '8' *i.e.* $\sqrt{64} = 8$

4. Find the square root of the following :

(a) 2304

The square root of the number 2304.

$\sqrt{2304} = 48$

	48
4	$\overline{23}\,\overline{04}$
	16
48	704
	704
	0

(b) 11664

The square root of the number 11664

$\sqrt{11664} = 108$

	108
1	$\overline{1}\,\overline{16}\,\overline{64}$
	1
208	1664
	1664
	0

(c) 529

The square root of the given number 529.

$\sqrt{529} = 23$

	23
2	$\overline{5}\,\overline{29}$
	4
43	129
	129
	0

(d) 625

The square root of the given number 625

$\sqrt{625} = 25$

	25
2	$\overline{6}\,\overline{25}$
	4
14	225
	225
	0

(e) 1444

The square root of the given number 1444

$\sqrt{1444} = 38$

	38
3	$\overline{14}\,\overline{44}$
	9
68	544
	544
	0

(f) 400

The square root of the given number 400

$\sqrt{400} = 20$

	20
2	$\overline{4}\,\overline{00}$
	4
40	0 0
	0

5. Find the least number that should divide the following to make it a perfect square :

(a) 1256

The factors of $1256 = 2 \times 2 \times 2 \times 157$

Since prime factors of 1256 are not in pairs, 1256 is not a perfect square.

To make it a perfect square, it should be divided by 314.

Hence, the required number is 314.

2	1256
2	628
2	314
157	157
	1

(b) 10220

The factors of $10220 = 2 \times 2 \times 5 \times 7 \times 73$

So, to make it a perfect square it should be divided by $5 \times 7 \times 73 = 255$

2	10220
2	5110
5	2555
7	511
73	73
	1

(c) 1452

The factors of $1452 = 2 \times 2 \times 3 \times 11 \times 11$

So, to make it a perfect square, it should be divided by 3.

2	1452
2	726
3	3635
11	121
11	11
	1

(d) 768

The factors of $768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

To make it a perfect square, so, it should be divided by ‘3’

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

6. Find the least number by which the following numbers should be multiplied to get a perfect square :

(a) 847

The factors of $847 = 7 \times 11 \times 11$

To make 847 a perfect square, we have to multiply it by 7.

7	847
11	121
11	11
	1

(b) 15488

The factors of $15488 = 4 \times 4 \times 4 \times 2 \times 11 \times 11$

To make 15488 a perfect square, we have to multiply it by 2.

4	15488
4	3872
4	968
2	242
11	121
11	11
	1

(c) 2028

The factors of $2028 = 2 \times 2 \times 3 \times 13 \times 13$

To make 2028 a perfect square, we have to multiply it by 3.

2	2028
2	1014
3	507
13	169
13	13
	1

(d) 1620

The factors of $1620 = 2 \times 2 \times 5 \times 9 \times 9$

To make 1620 a perfect square, we have to multiply it by 5.

4	1620
5	405
9	81
9	9
	1

7. What least number must be subtracted from the following numbers to make them a perfect square :

(a) 4931

31 is the smallest number which must be subtracted from 4931 to make it a perfect square.

	71
7	$\overline{49}\,\overline{31}$
	49
14	31

(b) 8934

98 is the smallest number which must be subtracted from 8934 to make it a perfect square.

	94
9	$\overline{89}\,\overline{34}$
	81
184	834
	736
	98

(c) 18255

30 is the smallest number which must be subtracted from 18255 to make it a perfect square.

	135
1	$\overline{1}\,\overline{82}\,\overline{55}$
	1
23	82
	69
265	1355
	1325
	30

8. What least number must be added to the following numbers to make them a perfect square :

(a) 92700

Because

$(304)^2 < 92700 < (305)^2$

Thus the least number must be added $= (305)^2 - 92700,$

$= 93025 - 92700 = 325$

	304
3	$\overline{9}\,\overline{27}\,\overline{00}$
	9
604	2700
	2416
	284

(b) 9598

Because $(97)^2 < 9598 < (98)^2$

Thus the least number must be added $= (98)^2 - 9598$

$= 9604 - 9598$

$= 6$

	97
9	$\overline{95}\,\overline{98}$
	81
187	1498
	1309
	189

(c) 54725

Because $(233)^3 < 54725 < (234)^2$

Thus, the least number must be added

$= (234)^2 - 54725$

$= 54756 - 54725 = 31$

	233
2	$\overline{5}\ \overline{47}\ \overline{25}$
	4
437	147
	129
463	1825
	1389
	436

9. Is 16224 a perfect square? If not, find the smallest number by which 16224 must be divided to make it a perfect square.

$$16224 = 4 \times 4 \times 2 \times 3 \times 13 \times 13$$

As the prime factor 2×3 or '6' is the left unpaired, 16224 is not a perfect square.

To make 16224 a perfect square we have to divide it by 6.

So the 6 is the required smallest number.

10. Find the smallest number by which 7436 must be multiplied to make it a perfect square. Also find the square root of the perfect square so obtained.

$$7436 = 2 \times 2 \times 13 \times 13 \times 11$$

As the prime factor 11 is left unpaired, 7436 is not a perfect square.

To make 7436 a perfect square, we have to multiply it by 11

so that the prime factor 11 gets paired

$$= 2 \times 2 \times 13 \times 13 \times 11 \times 11 = 81796$$

$\therefore$ Square root of 81796 is $\sqrt{81796} = 2 \times 13 \times 11 = 286.$

11. Find the least number exactly divisible by 6,9,15 and 20.

L.C.M. of 6, 9, 15, 20 is

$$3 \times 3 \times 2 \times 2 \times 5 = 180$$

So, the least number $= 180$

2	6, 9, 15, 20
3	3, 9, 15, 10
5	1, 3, 5, 10
	1, 3, 1, 2

12. Find the square root of the smallest square number divisible by 6, 8 and 15.

Solution : L.C.M. of 6, 8, 15 is

$$2 \times 2 \times 2 \times 3 \times 5 = 120$$

Since pime factors of 120 are not in pairs, 120 is not a perfect square. To make it a perfect square it should be multiplied by $2 \times 3 \times 5$.

Hence, the small square number is $120 \times 2 \times 3 \times 5 = 3600$

$\therefore$ $\sqrt{3600} = 60$

2	6, 8, 15
2	3, 4, 15
3	3, 2, 15
	1, 2, 5

13. Find the greatest five digit number that is a perfect square.

The greatest 5-digit number is 99999. Now we try to find whether 99999 is a perfect square. We get remainder '143' showing that 99999 is not a perfect square. Subtracting 143 from 99999, we can make it a perfect square.

i.e. $99999 - 143 = 99856$

$\therefore$ 99856 is the largest 5-digit number, which is a perfect square.

	316
3	$\overline{9}\ \overline{99}\ \overline{99}$
	9
61	99
	61
626	3899
	3756
	143

14. A general wishing to draw up his 64019 men in the form of a perfect square, found that he had 10 men over. Find the number of men in each row.

The men that could be that could be arranged in a square $= 64019 - 10$

$= 64009$

Therefore, Number of men in each row $= \sqrt{64009}$

$= 253.$

So, the number of men in each row = 253.

15. The area of a square park is equal to that of a rectangular field of length 96 m and breadth 54 m. Find the length of the side of the square.

Length of a rectangular = 96 m

and Breadth of a rectangular = 54 m

$\because$ Area of a rectangular = Length × Breadth

$= 96 \times 54 \text{ m}^2$

Let the length of the square = l

So the area of a square park = l^2

According to the question, $l^2 = 96 \times 54 \text{ m}^2$

$l^2 = 5184 \text{ m}^2$

or $l = \sqrt{5184} \text{ m}^2$

$\therefore$ $l = 72$ m

So, the length of a square field = 72 m

Exercise-3

1. Find the square root of the following :

(a) $\frac{64}{441} = \sqrt{\frac{64}{441}}$

$\sqrt{64} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2 \times 2 \times 2 = 8$

$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7} = 3 \times 7 = 21$

$\therefore$ $\sqrt{\frac{64}{441}} = \frac{8}{21}$

2	64
2	32
2	16
2	8
2	4
2	2
	1

3	441
3	147
7	49
7	7
	1

(b) $\frac{121}{625} = \sqrt{\frac{121}{625}}$

$\sqrt{121} = \sqrt{11 \times 11} = 11$

$\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5} = 5 \times 5 = 25$

$\therefore$ $\sqrt{\frac{121}{625}} = \frac{11}{25}$

11	121
11	11
	1

5	625
5	125
5	25
5	5
	1

(c) $\frac{3600}{289} = \sqrt{\frac{3600}{289}}$

$\sqrt{3600} = \sqrt{6 \times 6 \times 5 \times 5 \times 2 \times 2} = 6 \times 5 \times 2 = 60$

$\sqrt{289} = \sqrt{17 \times 17} = 17$

$\therefore$ $\sqrt{\frac{3600}{289}} = \frac{60}{17}$

6	3600
6	600
5	100
5	20
2	4
2	2
	1

17	289
17	17
	1

(d) $\frac{196}{361} = \sqrt{\frac{196}{361}}$

$\sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7} = 2 \times 7 = 14$

$\sqrt{361} = \sqrt{19 \times 19} = 19$

$\therefore \quad \sqrt{\frac{196}{361}} = \frac{14}{19}$

2	196
2	98
7	49
7	7
	1

19	361
19	19
	1

2. Evaluate the following :

(a) $\sqrt{5\frac{19}{25}} = \sqrt{\frac{144}{25}}$

$\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3} = 2 \times 2 \times 3 = 12$

$\sqrt{25} = \sqrt{5 \times 5} = 5$

$\therefore \quad \sqrt{\frac{144}{25}} = \frac{12}{5}$

5	25
5	5
	1

2	144
2	72
2	36
2	18
3	9
3	3
	1

(b) $\sqrt{6\frac{30}{289}} = \sqrt{\frac{1764}{289}}$

$\sqrt{1764} = \sqrt{2 \times 2 \times 3 \times 3 \times 7 \times 7} = 2 \times 3 \times 7 = 42$

$\sqrt{289} = \sqrt{17 \times 17} = 17$

$\therefore \quad \sqrt{\frac{1764}{289}} = \frac{42}{17}$

2	1769
2	882
3	441
3	147
7	49
7	7
	1

17	289
17	17
	1

(c) $\sqrt{5\frac{4}{9}} = \sqrt{\frac{49}{9}}$

$\sqrt{49} = \sqrt{7 \times 7} = 7$

$\sqrt{9} = \sqrt{3 \times 3} = 3$

$\therefore \quad \sqrt{\frac{49}{9}} = \frac{7}{3}$

7	49
7	7
	1

3	9
3	3
	1

(d) $\sqrt{\frac{625}{1296}}$

$\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5} = 5 \times 5 = 25$

$\sqrt{1296} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3} = 2 \times 2 \times 3 \times 3 = 36$

$\therefore \quad \sqrt{\frac{625}{1296}} = \frac{25}{36}$

5	625
5	125
5	25
5	5
	1

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

3. Find the square root of the following decimals, correct up to two decimal places :

(a) $33 \cdot 64$

$\therefore \quad \sqrt{33 \cdot 64} = 5 \cdot 8$

	$5 \cdot 8$
5	$\overline{33} \cdot \overline{64}$
	25
108	864
	864
	0

(b) $392 \cdot 64$

$\sqrt{392 \cdot 64} = 19 \cdot 8$ or $19 \cdot 81$

	$19 \cdot 8$
1	$3\,\overline{92} \cdot \overline{64}$
	1
29	292
	261
388	3164
	3104
	60

(c) $44 \cdot 1$

$\sqrt{44 \cdot 1} = 6 \cdot 64$

	$6 \cdot 64$
6	$\overline{44} \cdot \overline{10}\,\overline{00}$
	36
126	810
	756
132	5400
	5296
	104

(d) 5

$\sqrt{5} = 2 \cdot 23$

	$2 \cdot 23$
2	$5 \cdot \overline{00}\,\overline{00}$
	4
42	100
	84
443	1600
	1329
	271

4. Find the square root of $0 \cdot 9$ up to 3 decimal places.

	$0 \cdot 948$
0	$0 \cdot \overline{90}\,\overline{00}\,\overline{00}$
	0
9	90
	81
184	900
	736
1888	16400
	14784
	1616

$\therefore \quad \sqrt{0 \cdot 9} = 0 \cdot 948$

5. Find $\sqrt{0 \cdot 00053361}$

	$0 \cdot 0231$
0	$0 \cdot \overline{00}\,\overline{05}\,\overline{33}\,\overline{61}$
	0
0	00
	00
2	05
	04
43	133
	126
461	761
	461
	300

$\therefore \quad \sqrt{0 \cdot 00053361} = 0 \cdot 0231$

6. The area of a square field is $132 \cdot 25\ \text{km}^2$. If a man runs around the field three times, what is the distance travelled by him?

Solution : Area of square field $= 132 \cdot 25$ km

$\therefore \quad$ Length of a square field $= \sqrt{132 \cdot 25} = 11 \cdot 5$

$= 11 \cdot 5$ km

A man runs around the field 3 times

Distance travelled by the man $= 3 \times 11 \cdot 5$

$= 34 \cdot 5$ km

	$11 \cdot 5$
1	$1\overline{32} \cdot \overline{25}$
	1
21	32
	21
225	1125
	1125
	0

Chapter- 4 : Cubes and Cube Roots

Exercise-1

1. Find the cube of the following :

(a) -21

$(-21)^3 = (-21)\times(-21)\times(-21)$

$= -9261$

(b) 23

$(23)^3 = 23\times23\times23 = 12167$

(c) $3\cdot2$

$(3\cdot2)^3 = 3\cdot2\times3\cdot2\times3\cdot2 = 32\cdot768$

(d) $0\cdot6$

$(0\cdot6)^3 = 0\cdot6\times0\cdot6\times0\cdot6 = 0\cdot216$

2. Find the cube of the following rational numbers :

(a) $\frac{9}{10}$

$\left(\frac{9}{10}\right)^3 = \frac{9}{10}\times\frac{9}{10}\times\frac{9}{10} = \frac{729}{1000}$

(b) $\frac{-1}{8}$

$\left(\frac{-1}{8}\right)^3 = \frac{-1}{8}\times\frac{-1}{8}\times\frac{-1}{8} = \frac{-1}{512}$

(c) $\frac{27}{31}$

$\left(\frac{27}{31}\right)^3 = \frac{27}{31}\times\frac{27}{31}\times\frac{27}{31} = \frac{19683}{29791}$

(d) $2\frac{3}{11}$

$= \left(\frac{25}{11}\right)^3 = \frac{25}{11}\times\frac{25}{11}\times\frac{25}{11} = \frac{15625}{1331}$

3. Find the ones digit in the cube of :

(a) 444

In 444, the digit in ones place is 4 and also $(4)^3 = 64$, has 4 in the ones place, so the cube of 444 will have 4 in the ones place.

(b) 98

In 98, the digit in ones place is 8 and but $(8)^3 = 512$ has 2 in the ones place. So, the cube of 98 will have 2 in the ones place.

(c) 251

In 251, the digit in ones place is 1 and also $(1)^3 = 1$ has 1 in the ones place. So, the cube of 251 will have 1 in the ones place.

(d) 869

In 869, the digit in ones place is 9 and also $(9)^3 = 729$ has 9 in the ones place.

So, the cube of 869 will have 9 in the ones place.

4. Find the least number by which the following must be multiplied to make them a perfect cube :

(a) 14440

Here, $14400 = 2\times2\times2\times2\times2\times2\times5\times5\times3\times3$

The prime factors 5×5 and 3×3 do not form a group of three. So, 14400 is not a perfect cube. In order to make it a perfect cube so that we need one more 3 and 5 Hence, $3\times5 = 15$ must be multiplied to make it a perfect cube.

$14400\times15 = 2\times2\times2\times2\times2\times2\times5\times5\times5\times3\times3\times3$ which is a perfect cube.

So, the smallest number by which 14400 should be multiplied to get a perfect cube is 15.

2	14400
2	7200
2	3600
2	1800
2	900
2	450
5	225
5	45
3	9
3	3
	1

(b) 128

We have $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

One prime factor 2 does not appear in a group of three. So, 128 is not perfect cube. In order to make it a perfect cube, we need $2 \times 2 = 4$. Hence, 4 must be multiplied to make it a perfect cube.

$$128 \times 4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

which is a perfect cube.

So, we can say that the smallest number by which 128 should be multiplied to get a perfect cube is 4.

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(c) 864

We have $864 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

One prime factor 2 does not appear in a group of three. So, 864 is not a perfect cube. In order to make it a perfect cube, we need one more 2. Hence, 2 must be multiplied to make it a perfect cube $864 \times 2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ which is perfect cube.

So, we can say that the smallest number by which 864 should be multiplied to get perfect cube is 2.

2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(d) 26244

We have $26244 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

One prime factor $2 \times 3 = 6$ does not appear in a group of three. So, 26244 is not a perfect cue. In order to make it a perfect cube, we need one more 6. Hence 6 must be multiplied to make it a perfect cube.

$26244 \times 6 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times \times 3 \times 3 \times 3 \times 3 \times 3$ which is a perfect cube.

So, we can say that the smallest number by which 26244 should be multiplied to get perfect cube is 6.

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

5. Find the least number by which the following must be divided to make them a perfect cube :

(a) 392

We have $392 = 2 \times 2 \times 2 \times 7 \times 7$

The product of factors $7 \times 7 = 49$ which can not be grouped in a triplet is the smallest number by which when we divide 392, the quotient will be a perfect cube.

$392 \div 49 = 8$

Thus, $8 = 2 \times 2 \times 2$ is a perfect cube, so the required smallest number is 49.

2	392
2	196
2	98
7	49
7	7
	1

(b) 29160

We have $29160 = 2 \times 2 \times 2 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

The factor 5 can not be grouped in a triplet which is the smallest number by which if we divide 29160, the quotient will be a perfect cube.

$$29160 \div 5 = 5832$$

Thus, $5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ is a perfect cube, so the required smallest number is 5.

2	29160
2	14580
2	7290
5	3645
3	729
3	243
3	81
3	27
3	9
3	3
	1

(c) 8575

We have $8575 = 5 \times 5 \times 7 \times 7 \times 7$

5	8575
5	1715
7	343
7	49
7	7
	1

The product of factors $5 \times 5 = 25$ can not be grouped in a triplet which is the smallest number by which if we divide 8575, the quotient will be a perfect cube

$$8575 \div 25 = 343$$

Thus $343 = 7 \times 7 \times 7$ is a perfect cube, so the required smallest number is 25.

(d) 4374

We have $4374 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2	4374
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

The product of factors $2 \times 3 = 6$ can not be grouped in a triplet a which is the smallest number by which if we divide 4374, the quotient will be a perfect cube

$$4374 \div 6 = 729$$

Thus, $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ is a perfect cube, so the required smallest number is 6.

6. Express the following cubes as the sum of consecutive odd numbers :

(a) 7^3

$1^3 = 1$

$2^3 = 3 + 5$

$3^3 = 7 + 9 + 11$

$4^3 = 13 + 15 + 17 + 19$

$5^3 = 21 + 23 + 25 + 27 + 29$

$6^3 = 31 + 33 + 35 + 37 + 39 + 41$

$7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

(b) 8^3

$1^3 = 1$

$2^3 = 3 + 5$

$3^3 = 7 + 9 + 11$

$4^3 = 13 + 15 + 17 + 19$

$5^3 = 21 + 23 + 25 + 27 + 29$

$6^3 = 31 + 33 + 35 + 37 + 39 + 41$

$7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

(c) 10^3

$1^3 = 1$

$2^3 = 3 + 5$

$3^3 = 7 + 9 + 11$

$4^3 = 13 + 15 + 17 + 19$

$5^3 = 21 + 23 + 25 + 27 + 29$

$6^3 = 31 + 33 + 35 + 37 + 39 + 41$

$7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

$8^3 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

$9^3 = 73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89$

$10^3 = 91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109$

7. The length of a side of a cubical box is 15 cm. Find the volume of the box.

Length of a side of cubical box = 15 cm

Volume of the box = $(\text{length})^3$

$$= 15 \times 15 \times 15 = 3375 \text{ cm}^3$$

8. Find the smallest number by which 46305 is multiplied so as to make it a perfect cube.

We have $46305 = 5 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

The prime factor 5 does not appear in a group of three. So, 46305 is not a perfect cube. In order to make it a perfect cube, we need $5 \times 5 = 25$. Hence 46305 must be multiplied by 25 to make it a perfect cube.

5	46305
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$46305 \times 5 \times 5 = 5 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 25 = 1157625$$

which is a perfect cube. So, we can say that the smallest number by which 46305 should be multiplied to get a perfect cube is 25.

9. Find the value of the following by successive subtraction :

(a) $(11)^3 - (10)^3$ (b) $(6)^3 - (5)^3$

$$n^3 - (n-1)^3 = 1 + n(n-1) \times 3$$

$n^3 - (n-1)^3 = 1 + n(n-1) \times 3$

$(11)^3 - (11-1)^3 = (11)^3 - (10)^3$
$= 1 + 11 \times (11-1) \times 3$
$= 1 + 11 \times 10 \times 3$
$= 1 + 11 \times 30$
$= 1 + 330 = 331$

$(6)^3 - (6-1)^3 = 1 + 6 \times (6-1) \times 3$
$(6)^3 - (5)^3 = 1 + 6 \times 5 \times 3$
$= 1 + 90$
$= 91$

(c) $(21)^3 - (20)^3$

$n^3 - (n-1)^3 = 1 + n(n-1) \times 3$
$(21)^3 - (20)^3 = 1 + 21 \times (21-1) \times 3$
$= 1 + 21 \times 20 \times 3$
$= 1 + 21 \times 60$
$= 1 + 1260 = 1261$

Exercise-2

1. Find the cube root of the following by prime factorisation :

(a) 42875

$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$
$= 5^3 \times 7^3$
$= (5 \times 7)^3$
$= 35^3$
$\sqrt[3]{42875} = 35$

(b) 343

$343 = 7 \times 7 \times 7$
$= 7^3$
$\sqrt[3]{343} = 7$

(c) 6859

$6859 = 19 \times 19 \times 19$
$= 19^3$
$\sqrt[3]{6859} = 19$

(d) 15625

$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^3 \times 5^3$
$= (5 \times 5)^3$
$= (25)^3$
$\sqrt{15625} = 25$

3. Find the cube root of the following :

(a) $0 \cdot 512$

$$\sqrt[3]{0 \cdot 512} = \sqrt[3]{\frac{512}{1000}}$$

$$= \sqrt[3]{\frac{8 \times 8 \times 8}{10 \times 10 \times 10}}$$

$$= \frac{8}{10} = 0 \cdot 8$$

(b) $0 \cdot 004096$

$$\sqrt[3]{0 \cdot 004096} = \sqrt[3]{\frac{4096}{1000000}}$$

$$= \sqrt[3]{\frac{16 \times 16 \times 16}{100 \times 100 \times 100}}$$

$$= \frac{16}{100} = 0 \cdot 16$$

(c) $\frac{729}{1728}$

$$\frac{729}{1728} = \sqrt[3]{\frac{729}{1728}} = \sqrt[3]{\frac{9 \times 9 \times 9}{12 \times 12 \times 12}} = \frac{9}{12}$$

(d) $\frac{-343}{9261}$

$$\frac{-343}{9261} = \sqrt[3]{\frac{-343}{9261}}$$

$$= \sqrt[3]{\frac{-7 \times -7 \times -7}{21 \times 21 \times 21}} = \frac{-7}{21}$$

4. Find the cube root of the following :

(a) 144×96

$$\sqrt[3]{144 \times 96} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= 2 \times 2 \times 2 \times 3 = 24$$

(b) 45×75

$$= \sqrt[3]{45 \times 75} = \sqrt[3]{5 \times 3 \times 3 \times 5 \times 5 \times 3}$$

$$= 5 \times 3 = 15$$

(c) 125×3375

$$\sqrt[3]{125 \times 3375} = \sqrt[3]{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3}$$

$$= 5 \times 5 \times 3 = 75$$

(d) -216×729

$$\sqrt[3]{-216 \times 729} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$= 2 \times 3 \times 3 \times 3 = 54$$

5. Find the cube root of the following by estimation method :

(a) 12167

To estimate cube root by finding the units and tens digit of that number (cube root).

Step 1 : Since the units digits of given number is 7, the units digit of the cube root will be 343. Take the digit of ones place number that is 3.

Step 2 : Striking off the units, tens and hundreds digits of the given numbers, you will be left with the number 1 and the perfect cube less than 1 is 2.

Step 3 : The tens digit of the cube root of 12167 is 2.

Thus, the estimated cube root of 12167 is 23.

(b) 9261

To estimate cube root by finding the units and tens digits of that number (cube root).

Step 1 : Since the unit digit of given number is 1, the units digit of the cube root will be 1.

Step 2 : Striking off the units, tens and hundreds digit of the given number, you will be left with the number 9 and the perfect cube less than 9 is 2.

Step 3 : The tens digit of the cube root of 9261 is 1

Thus, the estimated cube root of 9261 is 21.

(c) 97336

To estimate cube root by finding the units and tens digit of that number (cube root).

Step 1: Since the units digit of given number is 6, the units digit of the cube root will be 6.

Step 2 : Striking off the units, tens and hundreds digit of the given number, you will be left with the number 9 and the perfect cube less than 9 is 4.

Step 3 : The tens digit of the cube root of 97336 is 4.

Thus, the estimated cube root of 97336 is 46.

(d) 592704

To estimate cube root by finding the units and tens digit of that number (cube root).

Step 1 : Since the units digit of given number is 4, the unit digit of the cube root will be 4.

Step 2 : Striking off the units, tens and hundred digits of the given number, you will be left with the number 5 and the perfect cube greater than 5 is 8.

Step 3 : The tens digit of the cube root of 592704 is 8.

Thus, the estimated cube root of 592704 is 84.

6. The total volume of two identical cubical containers is 3456 cm^3. Find the edge of one container.

The combined volume of two identical cubes is $= 3456\,cm^3$

One cubical container is half the volume of two cubical containers, so

$$= \frac{3456}{2}$$

$$= 1728\,cm^3$$

$\therefore$ One edge of container $= \sqrt[3]{1728} = 12\,cm$

7. What is the smallest number by which 4116 must be multiplied so that the product is a perfect cube? Find the cube root of the perfect cube so obtained.

Resolving 4116 as the product of prime factors

$$4116 = 2 \times 2 \times 3 \times 7 \times 7 \times 7$$

$\therefore$ In order to make 4116 a perfect cube, it must be multiplied by $2 \times 3 \times 3 = 18$

8. The volume of two cubes are 729 : 1331. Find the ratio of their edges. What is the ratio of their surface areas?

Let the edges be a and b of two cubes, then

$$\frac{a^3}{b^3} = \frac{729}{1331}$$

$$\left(\frac{a}{b}\right)^3 = \left(\frac{9}{11}\right)^3$$

or
$$\frac{a}{b} = \frac{9}{11}$$

The ratio of their surface areas $= \dfrac{6 \cdot (a)^2}{6 \cdot (b)^2} = \dfrac{6 \times (9)^2}{6 \times (11)^2} = \dfrac{81}{121}$

Chapter-5 : Linear Equations in One Variable

Exercise-1

1. Solve the following equations :

(a) $17x + 12 = 13x + 24$

Simplify both sides of the equality

$$17x + 12 = 13x + 24$$

Transpose all terms containing variable to L.H.S. and constant terms to R.H.S.

$$17x - 13x = 24 - 12$$

or $$4x = 12$$

$\therefore$ $$x = 3$$

(b) $2x + 3 = 7$

Simplify both sides of the equality

$$2x + 3 = 7$$

Transpose the term with variable to one side, we get

$$2x = 7 - 3$$

or $$2x = 4$$

$\therefore$ $$x = \frac{4}{2} = 2$$

(c) $\frac{x}{4} + 1 = -5$

$$\frac{x}{4} + 1 = -5$$

or $$\frac{x}{4} = -6$$

By cross-multiplication, we get

$$x = -6 \times 4$$
$$= -24$$

(d) $2a - 10 = -8$

Transposing the term with variable to one side, we get

$$2a = -8 + 10$$

or $$2a = 2$$

$\therefore$ $$a = 1$$

(e) $4(2x - 5) + 17 = 29$

$$8x - 20 + 17 = 29$$

Transposing the terms with variables to one side, we get

$$8x = 29 + 20 - 17$$
$$8x = 49 - 17$$

or $$8x = 32$$

$$x = \frac{32}{8} = 4$$

$\therefore$ $$x = 4$$

(f) $9(x-1) = 4(x-3)$

$$9x - 9 = 4x - 12$$

Transposing the terms with variables to one side, we get

$$9x - 4x = -12 + 9$$

or $$5x = -3$$

$\therefore$ $$x = \frac{-3}{5}$$

(g) $2m - \frac{2}{9} = 4m - \frac{4}{3}$

$$\frac{18m-2}{9} = \frac{12m-4}{3}$$

By cross-multiplication, we get

$$3(18m-2) = 9(12m-4)$$

$$54m - 6 = 108m - 36$$

Transposing the terms with variables to one side, we get

$$54m - 108m = -36 + 6$$

or $$-54m = -30$$

$$m = \frac{-30}{-54} = \frac{30}{54} = \frac{5}{9}$$

$\therefore$ $$m = \frac{5}{9}$$

(h) $0\cdot 5x + 1\cdot 5 = 0\cdot 7x - 0\cdot 9$

Transposing the terms with variables to one side, we get

$$0\cdot 5x - 0\cdot 7x = -0\cdot 9 - 1\cdot 5$$

$$-0\cdot 2x = -2\cdot 4$$

$$0\cdot 2x = 2\cdot 4$$

$$x = \frac{2\cdot 4}{0\cdot 2} = 12$$

$\therefore$ $$x = 12$$

2. Solve the following equations and verify your answers :

(a) $\frac{x}{2} - \frac{x}{3} = 8$

$$\frac{3x - 2x}{6} = 8$$

or $$\frac{x}{6} = 8$$

By cross-multiplication, we get

$$x = 8 \times 6$$

$$= 48$$

Checking : Substitate the value of $x = 48$ in the equation.

L.H.S. $= \frac{x}{2} - \frac{x}{3}$

$$= \frac{48}{2} - \frac{48}{3}$$

$$= 24 - 16 = 8$$

R.H.S. = 8

∴ Hence verified.

(b) $\frac{3x+2}{4} = 8$

By cross-multiplication, we get

$$3x + 2 = 8 \times 4$$

or $\quad 3x + 2 = 32$

or $\quad 3x = 32 - 2$

$$3x = 30$$

or $\quad x = \frac{30}{3}$

∴ $\quad x = 10$

Checking : Substitute the value of $x = 10$ in the equation.

L.H.S. $= \frac{3x+2}{4}$

$$= \frac{3 \times 10 + 2}{4} = \frac{30+2}{4} = \frac{32}{4} = 8$$

R.H.S. = 8

∴ Hence verified.

(c) $\frac{x+2}{6} + \frac{x-3}{3} = x$

$$\frac{(x+2) + 2\,(x-3)}{6} = x$$

By cross-multiplication, we get

$$(x+2) + 2\,(x-3) = 6x$$

or $\quad x + 2 + 2x - 6 = 6x$

$$3x - 4 = 6x$$

Transposing the terms with variables to one side, we get

$$-4 = 6x - 3x$$

or $\quad -4 = 3x$

∴ $\quad x = \frac{-4}{3}$

Checking : Substitute the value of $x = \frac{-4}{3}$ in the equation

L.H.S. $= \frac{x+2}{6} + \frac{x-3}{3}$

$$= \frac{\frac{-4}{3} + 2}{6} + \frac{\frac{-4}{3} - 3}{3}$$

$$= \frac{\frac{-4+6}{3}}{6} + \frac{\frac{-4-9}{3}}{3}$$

$$= \frac{2}{18} + \frac{(-13)}{9}$$

$$= \frac{2}{18} - \frac{13}{9}$$

$$= \frac{2-26}{18} = \frac{-24}{18} = \frac{-4}{3}$$

R.H.S. $= \frac{-4}{3}$

$\therefore$ Hence verified.

(d) $\frac{(3y-1)}{4} + \frac{(2y+3)}{3} = \frac{1-7y}{6}$

$$\frac{3(3y-1)+4\,(2y+3)}{12} = \frac{1-7y}{6}$$

By cross-multiplication, we get

$$6[3\,(3y-1)+4\,(2y+3)] = 12\,(1-7y)$$

$$6\,[9y-3+8y+12] = 12-84y$$

$$54y-18+48y+72 = 12-84y$$

Transposing the terms with variables to one side, we get

$$54y+48y+84y = 12-72+18$$

$$186y = 30-72$$

$$186y = -42$$

$$y = \frac{-42}{186} = \frac{-7}{31}$$

$\therefore$ $y = \frac{-7}{31}$

Checking : Substitute the value of $y = \frac{-7}{31}$ in the equation

L.H.S. $\frac{(3y-1)}{4} + \frac{(2y+3)}{3}$

$$= \frac{3\times\left(\frac{-7}{31}\right)-1}{4} + \frac{2\times\left(\frac{-7}{31}\right)+3}{3}$$

$$= \frac{\frac{-21}{31}-1}{4} + \frac{\frac{-14}{31}+3}{3}$$

$$= \frac{\frac{-21-31}{31}}{4} + \frac{\frac{-14+93}{31}}{3}$$

$$= \frac{-52}{124} + \frac{79}{93}$$

$$= \frac{-156+316}{372} = \frac{160}{372} = \frac{40}{93}$$

$$\text{R.H.S.} = \frac{1-7y}{6} = \frac{1-7\times\left(\frac{-7}{31}\right)}{6}$$

$$= \frac{1+\frac{49}{31}}{6} = \frac{\frac{31+49}{31}}{6} = \frac{80}{186} = \frac{40}{93}$$

∴ Hence verified.

(e) $\frac{(3x+1)}{16} + \frac{(2x-3)}{7} = \frac{(x+3)}{8} + \frac{(3x-1)}{14}$

or $\frac{7\,(3x+1)+16\,(2x-3)}{112} = \frac{7\,(x+3)+4\,(3x-1)}{56}$

By cross-multiplication, we get

$$\frac{(21x+7+32x-48)}{112} = \frac{(7x+21+12x-4)}{56}$$

$$56\,(21x+7+32x-48) = 112\,(7x+21+12x-4)$$

$$7\,(21x+7+32x-48) = 14\,(7x+21+12x-4)$$

$$21x+7+32x-48 = 2\,(7x+21+12x-4)$$

$$53x-41 = 14x+42+24x-8$$

$$53x-41 = 38x+34$$

$$53x-38x = 34+41$$

$$15x = 75$$

$$x = \frac{75}{15} = 5$$

∴ $x = 5$

Checking : Substitute the value of $x = 5$ in the equation

$$\text{L.H.S.} = \frac{(3x+1)}{16} + \frac{(2x-3)}{7}$$

$$= \frac{(3\times5+1)}{16} + \frac{(2\times5-3)}{7}$$

$$= \frac{15+1}{16} + \frac{10-3}{7} = \frac{16}{16} + \frac{7}{7}$$

$$= 1+1 = 2$$

$$\text{R.H.S.} = \frac{(x+3)}{8} + \frac{(3x-1)}{14}$$

$$= \frac{5+3}{8} + \frac{3\times5-1}{14}$$

$$= \frac{8}{8} + \frac{15-1}{14} = \frac{8}{8} + \frac{15-1}{14}$$

$$= \frac{8}{8} + \frac{14}{14}$$

$$= 1+1 = 2$$

L.H.S. = R.H.S.

Hence verified.

(f) $8x + 4 = 3(x - 1) + 7$

$$8x + 4 = 3x - 3 + 7$$

Transposing the terms with variables to one side, we get

$$8x - 3x = -3 + 7 - 4$$

or $$5x = 7 - 7$$

$$5x = 0$$

$\therefore$ $$x = 0$$

Checking : Substitute the value of $x = 0$ in the equation

L.H.S. $= 8x + 4$

$$= 8 \times 0 + 4$$
$$= 0 + 4$$
$$= 4$$

R.H.S. $= 3(x - 1) + 7$

$$= 3(0 - 1) + 7$$
$$= 3 \times 0 - 3 + 7$$
$$= 0 - 3 + 7$$
$$= 4$$

L.H.S. = R.H.S.

$\therefore$ Hence verified.

(g) $\dfrac{3x + 4}{2 - 6x} = \dfrac{-2}{5}$

By cross-multiplication, we get

$$5(3x + 4) = -2(2 - 6x)$$
$$15x + 20 = -4 + 12x$$

Transposing the terms with variables to one side, we have

$$15x - 12x = -4 - 20$$
$$3x = -24$$
$$x = \frac{-24}{3} = -8$$

$\therefore$ $$x = -8$$

Checking : Substitute the value of $x = -8$ in the equation

L.H.S. $= \dfrac{3x + 4}{2 - 6x}$

$$= \frac{3 \times (-8) + 4}{2 - 6(-8)} = \frac{-24 + 4}{2 + 48}$$
$$= \frac{-20}{50} = \frac{-2}{5}$$

R.H.S. $= \dfrac{-2}{5}$

L.H.S. = R.H.S.

$\therefore$ Hence verified.

(h) $0 \cdot 6x - 0 \cdot 28x + 0 \cdot 8 = 1 \cdot 16$

Transposing the terms with variables to one side, we get

$$0\cdot 6x - 0\cdot 28x = 1\cdot 16 - 0\cdot 8$$

$$0\cdot 32x = 0\cdot 36$$

or $$x = \frac{0\cdot 36}{0\cdot 32} = \frac{9}{8}$$

∴ $$x = \frac{9}{8}$$

Checking : Substitute the value of $x = \frac{9}{8}$ in the equation

$$\text{L.H.S.} = 0\cdot 6x - 0\cdot 28x + 0\cdot 8$$

$$= 0\cdot 6 \times \frac{9}{8} - 0\cdot 28 \times \frac{9}{8} + \frac{8}{10}$$

$$= \frac{6}{10} \times \frac{9}{8} - \frac{28}{100} \times \frac{9}{8} + \frac{8}{10}$$

$$= 0\cdot 675 - 0\cdot 315 + 0\cdot 8 = 1\cdot 16$$

R.H.S. $= 1\cdot 16$ ∴ L.H.S. = R.H.S.

Hence veified.

(i) $\frac{1}{4}x - 1 = 2x$

Transposing the terms with variables to one side, we get

$$\frac{1}{4}x - 2x = 1$$

$$\frac{x - 8x}{4} = 1$$

By cross-multiplication, we get

$$x - 8x = 4$$

or $$-7x = 4$$

∴ $$x = \frac{-4}{7}$$

Checking : Substitute the value of $x = \frac{-4}{7}$ in the equation

L.H.S. $= \frac{1}{4} \times \left(\frac{-4}{7}\right) - 1$

$$= \frac{-1}{7} - 1$$

$$= \frac{-1-7}{7} = \frac{-8}{7}$$

R.H.S. $= 2x = 2 \times \frac{-4}{7} = \frac{-8}{7}$

L.H.S. = R.H.S.

∴ Hence verified.

(j) $m - \frac{(m-1)}{2} = 1 - \frac{(m-2)}{3}$

or $$\frac{2m-(m-1)}{2}=\frac{3-(m-2)}{3}$$

By cross-multiplication, we get

$$3\,[2m-(m-1)]=2\,[3-(m-2)]$$
$$6m-3m+3=6-2m+4$$

Transposing the terms with variables to one side, we get

$$6m-3m+2m=6+4-3$$

or $$8m-3m=7 \text{ or } 5m=7$$

∴ $$m=\frac{7}{5}$$

Checking : Substitute the value of $m=\frac{7}{5}$ in the equation

L.H.S. $= m-\frac{m-1}{2}$

$$=\frac{7}{5}-\frac{\left(\frac{7}{5}-1\right)}{2}$$
$$=\frac{7}{5}-\frac{\left(\frac{7-5}{5}\right)}{2}$$
$$=\frac{7}{5}-\frac{2}{10}$$
$$=\frac{7}{5}-\frac{1}{5}$$
$$=\frac{7-1}{5}$$
$$=\frac{6}{5}$$

Now put the value $m=\frac{7}{5}$ in R.H.S. $=1-\frac{(m-2)}{3}$

$$=1-\frac{\left(\frac{7}{5}-2\right)}{3}$$
$$=1-\frac{\frac{(7-10)}{5}}{3}$$
$$=1-\frac{(-3)}{15}$$
$$=1+\frac{1}{5}$$
$$=\frac{5+1}{5}=\frac{6}{5}$$

∴ R.H.S. = L.H.S. Hence verified.

(k) $\frac{2x}{5} - \frac{5x}{3} = \frac{1}{15}$

$$\frac{6x - 25x}{15} = \frac{1}{15}$$

or $$\frac{-19x}{15} = \frac{1}{15}$$

or $$-19x = 1$$

∴ $$x = \frac{-1}{19}$$

Checking : Substitute the value of $x = \frac{-1}{19}$ in the equation

$$\text{L.H.S.} = \frac{2x}{5} - \frac{5x}{3} = \frac{2 \times \left(-\frac{1}{19}\right)}{5} - \frac{5 \times \left(\frac{-1}{19}\right)}{3}$$

$$= \frac{-2}{19 \times 5} + \frac{5}{19 \times 3}$$

$$= \frac{-2 \times 3 + 5 \times 5}{19 \times 5 \times 3}$$

$$= \frac{-6 + 25}{19 \times 5 \times 3} = \frac{19}{19 \times 15} = \frac{1}{15}$$

L.H.S. = R.H.S.

∴ Hence verified.

Exercise-2

1. Solve the following equations :

(a) $\frac{9x - 7}{3x + 5} = \frac{3x - 4}{x + 6}$

By cross-multiplication, we get

$$(9x - 7)(x + 6) = (3x - 4)(3x + 5)$$

or $$9x^2 + 54x - 7x - 42 = 9x^2 + 15x - 12x - 20$$

or $$9x^2 + 47x - 42 = 9x^2 + 3x - 20$$

$$47x - 42 = 3x - 20$$

$$47x - 3x = -20 + 42$$

$$44x = 22$$

or $$x = \frac{22}{44} = \frac{1}{2}$$

∴ $$x = \frac{1}{2}$$

(b) $\frac{1}{x - 1} + \frac{3}{x + 1} = \frac{4}{x}$

Taking L.C.M.,

$$\frac{(x+1)+3(x-1)}{(x-1)(x+1)}=\frac{4}{x}$$

$$\frac{x+1+3x-3}{(x-1)(x+1)}=\frac{4}{x}$$

By cross-multiplication,

$$x^2+x+3x^2-3x=4(x^2-1)$$

$$4x^2-2x=4x^2-4$$

By transposing the terms, we get

$$4x^2-2x-4x^2=-4$$

$$-2x=-4$$

$$x=\frac{-4}{-2}$$

$\therefore$ $$x=2$$

(c) $.\frac{4z-3}{5}+\frac{1}{2}=\frac{3z+4}{4}-2$

$$\frac{2(4z-3)+5}{10}=\frac{3z+4-8}{4}$$

or $$\frac{8z-6+5}{10}=\frac{3z-4}{4}$$

By cross-multiplication, we get

$$4(8z-1)=10(3z-4)$$

$$16z-2=15z-20$$

$$16z-15z=-20+2$$

$\therefore$ $$z=-18$$

(d) $\frac{4x-5}{6x+3}=\frac{2x-5}{3x-2}$

By cross-multiplication, we get

$$(4x-5)(3x-2)=(2x-5)(6x+3)$$

$$12x^2-8x-15x-10=12x^2+6x-30x-15$$

$$-23x+10=-24x-15$$

$$-23x+24x=-15-10$$

$\therefore$ $$x=-25$$

(e) $\frac{(3x+1)}{2}+\frac{(2x+5)}{3}=26$

$$\frac{3(3x+1)+2(2x+5)}{6}=26$$

or $$\frac{9x+3+4x+10}{6}=26$$

$$\frac{13x+13}{6}=26$$

By cross-multiplication, we get

$$13x+13=26\times 6$$

or $$13x + 13 = 156$$
$$13x = 156 - 13$$
$$13x = 143$$
or $$x = \frac{143}{13} = 11$$
$\therefore$ $$x = 11$$

(f) $\frac{4x+8}{5x+8} = \frac{5}{6}$

By cross-multiplication, we get
$$6\,(4x+8) = 5\,(5x+8)$$
or $$24x + 48 = 25x + 40$$
$$25x - 24x = 48 - 40$$
$\therefore$ $$x = 8$$

(g) $\frac{0 \cdot 4z - 3}{1 \cdot 5z + 9} = \frac{-7}{5}$

By cross-multiplication, we get
$$5\,(0 \cdot 4z - 3) = -\,7\,(1 \cdot 5z + 9)$$
$$2 \cdot 0z - 15 = -10 \cdot 5z - 63$$
or $$2 \cdot 0z + 10 \cdot 5z = -\,63 + 15$$
$$12 \cdot 5z = -\,48$$
or $$z = \frac{-48}{12 \cdot 5} = \frac{-480}{125}$$
$\therefore$ $$z = -\frac{96}{25}$$

(h) $18y + 3y - \frac{3}{5} = 21 + 5y - 2y$
$$21y - \frac{3}{5} = 21 + 3y$$
or $$\frac{105y - 3}{5} = 21 + 3y$$
By cross-multiplication, we get
$$105y - 3 = 105 + 15y$$
$$105y - 15y = 105 + 3$$
$$90y = 108$$
or $$y = \frac{108}{90} = \frac{54}{45} = \frac{6}{5}$$
$\therefore$ $$y = \frac{6}{5}$$

(i) $\frac{3t-2}{3} + \frac{2t+3}{3} = t + \frac{7}{6}$
$$\frac{3t - 2 + 2t + 3}{3} = \frac{6t + 7}{6}$$

or $$\frac{5t+1}{3} = \frac{6t+7}{6}$$

$$5t+1 = \frac{6t+7}{2}$$

By cross multiplication, we get

$$10t+2 = 6t+7$$

$$10t-6t = 7-2$$

or $$4t = 5$$

$\therefore$ $$t = \frac{5}{4}$$

Exercise-3

1. The sum of two numbers is 43. Their difference is 13. Find the numbers.

Let the two numbers be x and y

According to question,

$$x + y = 43 \quad .... \text{(i)}$$

$$x - y = 3 \quad .... \text{(ii)}$$

Adding equations (i)and (ii)

$$\begin{array}{r} x + y = 43 \\ + \; x - y = 13 \\ \hline 2x = 56 \end{array} \quad ... \text{(iii)}$$

or $$x = \frac{56}{2}$$

$\therefore$ $$x = 28$$

Putting the value of x in eqn. (i)

we get $$x + y = 43$$

$$28 + y = 43$$

or $$y = 43 - 28$$

$$y = 15$$

$\therefore$ Numbers are 28 and 15.

2. The sum of three consecutive multiples of 8 is 888. Find the multiples.

We assume that the three consecutive multiples of 8 are $8x$, $8x+8$, $8x+16$

Now according to question,

$$8x + (8x+8) + (8x+16) = 888$$

$$24x + 24 = 888$$

$$24x = 888 - 24$$

$$24x = 864$$

or $$x = \frac{864}{24}$$

$\therefore$ $$x = 36$$

So, the three required multiples of 8 are

$$8x = 8 \times 36$$

$$= 288$$

$$8x+8=8\times36+8$$
$$=288+8$$
$$=296$$
$$8x+16=8\times36+16$$
$$=288+16$$
$$=304$$

$\therefore$ The required numbers are 288, 296 and 304.

3. Seven more than half of a number is 42. Find the number.

Let the number be x.

According to question,

$$7+\frac{x}{2}=42$$

or $$\frac{14+x}{2}=42$$

$$14+x=42\times2$$
$$14+x=84$$

or $$x=84-14$$

$\therefore$ $$x=70$$

4. Two angles of a right angled triangle are in the ratio 1 : 5. Find each angle of the triangle.

As the triangle is right angled, the value of an angle is 90°. So, the sum of other two angles $=180-90=90$ which are given to be in the ratio 1 : 5, so the two parts $=\frac{1}{6},\frac{5}{6}$ (by sum of the numbers, the denominator is formed)

Smaller angle $=\frac{1}{6}$ of $90°=\frac{1}{6}\times90°=15°$

Other angle $=\frac{5}{6}$ of $90°=\frac{5}{6}\times90°=75°$

Therefore angles are 15°, 75°.

5. Three numbers are in the ratio 4 : 5 : 7. If the sum of the largest and the smallest number is 72 more than the third number, find the numbers.

The ratio of three numbers = 4 : 5 : 7

Then the numbers will be $4x$, $5x$ and $7x$

Now, the aaccording to question, the largest number = $7x$

The smallest number = $4x$

$$7x+4x=72+5x$$

or $$11x=72+5x$$

$$11x-5x=72$$
$$6x=72$$

or $$x=\frac{72}{6}$$

$\therefore$ $$x=12$$

Hence the numbers

$$4x=4\times12$$
$$=48$$

$$5x = 5 \times 12$$
$$= 60$$
and $$7x = 7 \times 12$$
$$= 84$$

$\therefore$ Numbers are 48, 60 and 84.

6. Find three consecutive odd numbers whose sum is 45.

The three consecutive odd numbers are x, $x+2$, $x+4$

According to question,

$$x + (x+2) + (x+4) = 45$$
$$3x + 6 = 45$$
$$3x = 45 - 6$$
$$3x = 39$$
or $$x = \frac{39}{3}$$
$\therefore$ $$x = 13$$

Hence the three consecutive numbers are :

$$x = 13$$
$$x + 2 = 13 + 2$$
$$= 15$$
and $$x + 4 = 13 + 4$$
$$= 17$$

So, numbes are 13, 15, 17.

7. The age of Rashmi and Ravi are in the ratio 5 : 3. Five years hence, the ratio of their ages will be 10 : 7. Find their present ages.

Let the ages of Rashmi and Ravi be $5x$ and $3x$, then according to question,

Five years hence, the ratio of their ages $= 10 : 7$

$$\frac{5x+5}{3x+5} = \frac{10}{7}$$

By cross-multipication,

$$35x + 35 = 30 + 50$$
$$35x - 30x = 50 - 35$$
$$5x = 15$$
$\therefore$ $$x = 3$$

Hence the present ages of Rashmi

$$= 5x = 5 \times 3 = 15 \text{ years}$$

The present age of Ravi $3x = 3 \times 3 = 9$ years

8. Amitabh travelled $\frac{2}{5}$ of his journey by train, $\frac{1}{3}$ by taxi, $\frac{1}{6}$ by bus and remaining 6 km on foot. What is the length of his journey?

Let the total length $= x$ km

So, according to the question,

Journey by train $= \frac{2x}{5}$

Journey by taxi = $\frac{x}{3}$

Journey by bus = $\frac{x}{6}$

Journey by foot = 6 km

Total length of journey $\quad x = \frac{2x}{5} + \frac{x}{3} + \frac{x}{6} + 6$

$$x = x\left(\frac{2}{5} + \frac{1}{3} + \frac{1}{6}\right) + 6$$

$$x = x\left(\frac{6\times 2 + 10 + 5\times 1}{30}\right) + 6$$

$$x = x\left(\frac{12 + 10 + 5}{30}\right) + 6$$

$$x = x\left(\frac{27}{30}\right) + 6$$

$$x = \frac{27x}{30} + 6$$

$$x = \frac{9x}{10} + 6$$

$$x - \frac{9x}{10} = 6$$

Taking L.C.M. of L.H.S.,

$$\frac{10x - 9x}{10} = 6$$

By cross-multipication,

$$x = 10\times 6$$
$$= 60$$

$\therefore$ Total length = 60 km

9. Three numbers are in the ratio 3 : 4 : 5. If the sum of the largest and the smallest numbers exceeds the third number by 64, find the numbers.

The three numbers are in the ratio 3 : 4 : 5

So, First number = $3x$

Second number = $4x$

Third number = $5x$

According the question,

$$3x + 5x = 4x + 64$$

or $\quad 8x = 4x + 64$

$$8x - 4x = 64$$

or $\quad 4x = 64$

$\therefore \quad x = \frac{64}{4}$

$$= 16$$

So, the three numbers are

$$3x = 3 \times 16 = 48$$
$$4x = 4 \times 6 = 64$$
$$5x = 5 \times 16 = 80$$

$\therefore$ Numbers are 48, 64 and 80.

10. In a test, Gaurav obtained 9 marks more than Paurav whereas Ravi obtained 5 marks more than the marks obtained by Paurav and Gaurav. If all of them together obtained 195 marks, find the marks obtained by each of them.

According to question,

Let the marks obtained by Paurav $= x$

$\therefore$ Gaurav obtained marks $= x + 9$

Then, Ravi obtained marks $= x + (x + 9) + 5$

$= 2x + 14$

According to the condition,

all of them together obtained marks $= 195$ marks

Gaurav + Paurav + Ravi $= 195$

$$x + (x + 9) + 2x + 14 = 195$$
$$4x + 23 = 195$$

or $$4x = 195 - 23$$
$$4x = 172$$

or $$x = \frac{172}{4}$$

$\therefore$ $$x = 43$$

Hence, Marks obtained by

Paurav = 43

Gaurav $= x + 9$ $= 43 + 9 = 52$

Ravi $= 2x + 14$ $= 2 \times 43 + 14$

$= 100$

11. The denominator of a fraction exceeds its numerator by 4. If the numerator and denominator are both increased by 3, the new fraction becomes $\frac{4}{5}$. Find the original fraction.

Let the number be $= \frac{x}{x + 4}$

According to the question,

$$\frac{x + 3}{x + 4 + 3} = \frac{4}{5}$$
$$\frac{x + 3}{x + 7} = \frac{4}{5}$$

By cross-multiplication,

$$5(x + 3) = 4(x + 7)$$
$$5x + 15 = 4x + 28$$
$$5x - 4x = 28 - 15$$

$\therefore$ $$x = 13$$

Thus, the original fraction $= \frac{13}{13+4}$

$= \frac{13}{17}$

12. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. How old are they now?

Let the present age of son = x years

So, the present age of father = $7x$ years

Two years ago :

Son's age = $(x-2)$ years

Father's age = $(7x-2)$ years

According to the question,

$$(7x-2) = 13\,(x-2)$$

or $$7x-2 = 13x-26$$

or $$13x - 7x = 26-2$$

or $$6x = 24$$

$$x = \frac{24}{6}$$

∴ $$x = 4$$

Thus, son's present age = 4 years

Father's present age = $7 \times 4 = 28$ years

13. Ramesh's father's present age is 3 times Ramesh's age. After 12 years, his father's age will be twice his age. Find their present ages.

Let the Ramesh's present age = x years

So, the father's present age = $3x$ years

After 12 years

Ramesh's age = $x+12$ years

Father's age = $3x+12$ years

According to the question,

$$3x+12 = 2\,(x+12)$$

$$3x+12 = 2x+24$$

or $$3x-2x = 24-12$$

∴ $$x = 12$$

Thus, the present age of Ramesh = 12 years

Present age of father = $3 \times 12 = 36$ years

14. The ratio of the ages of Radha and Rahul two years ago was 2 : 3. Four years from now, the ratio of their ages will be 3 : 4. Find their present ages.

Let he present age of Radha = x years

and the present age of Rahul = y years

According to th question,

Two years ago, ratio of their ages

$$\frac{x-2}{y-2} = \frac{2}{3} \text{ or } 3\,(x-2) = 2\,(4-2)$$

$$3x-6 = 2y-4$$

$$3x - 2y = -4 + 6$$

or $$3x - 2y = 2 \quad ... (i)$$

Now, four years hence the ratio

$$\frac{x+4}{y+4} = \frac{3}{4}$$

$$4(x+4) = 3(y+4)$$

$$4x + 16 = 3y + 12$$

$$4x - 3y = 12 - 16$$

or $$4x - 3y = -4 \quad ... (ii)$$

Now Multiplying eqn. (i) by 3 and eqn. (ii) by 2, we, get

$$9x - 6y = 6 \quad ... (iii)$$

$$8x - 6y = -8 \quad ... (iv)$$

Subtracting eqn. (iii) from eqn (iv),

$$(9x - 6y) - (8x - 6y) = 6 - (-8)$$

$$9x - 6y - 8x + 6y = 6 + 8$$

∴ $$x = 14$$

Now putting the value x in eqn. (i),

$$3 \times 14 - 2y = 2$$

$$42 - 2y = 2$$

$$-2y = 2 - 42$$

or $$-2y = -40$$

∴ $$y = \frac{-40}{-2}$$

$$= 20$$

Thus, the present age of Radha = 14 years

and the present age of Rahul = 20 years

15. The sum of the present ages of a father and son is 53 years. Four years ago, the father's age was four times the age of the son. Find their present ages.

Let the present age of father = x years

the present age of son = y years

According the question,

$$x + y = 53 \quad ... (i)$$

and four years ago :

Father's age = $x - 4$ years

Son's age = $y - 4$ years

$$x - 4 = 4(y - 4)$$

$$x - 4 = 4y - 16$$

$$x - 4y = -16 + 4$$

$$x - 4y = -12 \quad ...(ii)$$

Now subtracting eqn (ii) from eqn (i)

$$(x + y) - (x - 4y) = 53 - (-12)$$

$$x + y - x + 4y = 53 + 12$$

$$5y = 65$$

or $y = \frac{65}{5}$

$\therefore$ $y = 13$

Now putting the value of y in eqn. (i),

$$x + y = 53$$
$$x + 13 = 53$$

or $x = 53 - 13$

$= 40$

$\therefore$ The father's present age = 40 years

and the son's present age = 13 years

16. The length of a rectangle exceeds its breadth by 7 cm. If the length is decreased by 4 cm and the breadth is increased by 3 cm, the area of the new rectangle is the same as the area of the original rectangle. Find the length and breadth of the original rectangle.

Let the breadth of a rectangle = x cm

Length of a rectangle = $(x + 7)$ cm

New breath = $(x + 3)$ cm

and New length = $x + 7 - 4 = (x + 3)$ cm

According to question,

Area of new rectangle = Area of the original rectangle

[$\because$ Area of rectangle = $l \times b$]

So, $(x + 3) \times (x + 3) = (x + 7) \times x$

$$x^2 + 3x + 3x + 9 = x^2 + 7x$$
$$x^2 - x^2 + 9 = 7x - 6x$$

$\therefore$ $x = 9$

The breadth 'b' = 9 cm

Length 'l' = $x + 7$ $= 9 + 7 = 16$ cm

17. The pocket money with Nirmal and Nitesh are in the ratio 5 : 7. If the pocket money of both of them is increased by Rs. 225, the new ratio will be 3 : 4. Find the pocket money they had.

Let the pocket money of Nirmal = $5x$

the pocket money of Nitesh = $7x$

After increasing Rs. 225,

Nirmal's pocket money = $5x + 225$

and Nitesh's pocket money = $7x + 225$

$\therefore$ $\frac{5x + 225}{7x + 225} = \frac{3}{4}$

By cross-multiplication,

$$4(5x + 225) = 3(7x + 225)$$
$$20x + 900 = 21x + 675$$
$$20x - 21x = 675 - 900$$
$$-x = -225$$

$\therefore$ $x = 225$

Now the pocket money of

Nirmal = $5 \times 225 =$ ₹1125

Nitesh = $7 \times 225 =$ ₹1575

18. A number consists of 2 digits whose sum is 8. If 18 is added to the number, the digits interchange their places. Find the number.

Let the units digit of the original number be $= x$

Then the tens digit of the original number $= 8 - x$

The original number in expanded notation $= 10\,(8 - x) + 1 \times x$

$= 80 - 10x + x$

$= 80 - 9x$

On interchanging the digits, the unit digit $= 8 - x$

and the tens digit $= x$

$\therefore$ The number obtained by interchanging the digits in expanded notation.

$= 10x + (8 - x) \times 1$

$= 10x + 8 - x$

$= 9x + 8$

According to question,

$$80 - 9x + 18 = 9x + 8$$
$$98 - 9x = 9x + 8$$
$$-9x - 9x = 8 - 98$$

or $$-18x = -90$$

$\therefore$ $$x = 5$$

The units digit of the required number $= 5$

The tens digit $= 8 - x = 8 - 5 = 3$

So, The original number $= 35$

19. Rs. 1260 is divided into two parts. The first part is 4 more than 3 times the second part. Find the two parts.

Let first part be x

$\therefore$ Second part $= 1260 - x$

According to the question,

$$x + 4 = 3\,(1260 - x)$$
$$x + 4 = 3780 - 3x$$
$$x + 3x = 3780 - 4$$

or $$4x = 3776$$

$\therefore$ $$x = 944$$

First part is 944, second part is $1260 - 944 = 316$

20. The boat goes downstream and covers a distance in 5 hours, while it covers the same distance upstream in 8 hours. If the speed of the stream is 3 km/h, find the speed of the boat in still water.

Let the speed of the boat in till water be x km/h.

Speed of water or stream = 3 km/h

Speed of boat downstream $= (x + 3)$ km/h

Distance covered in 5 hours = speed $\times$ time

$= 5 \times (x + 3)$ km

Speed of the boat upstream $= (x - 3)$ km/h

Distance covered in 8 hours = speed $\times$ time

$= 8 \times (x - 3)$ km

But the boat covers the same distance upstream and downstream. Therefore,

$$8\,(x - 3) = 5\,(x + 3)$$

or $8x - 24 = 5x + 15$

$8x - 5x = 15 + 24$

or $3x = 39$ or $x = \frac{39}{3}$

$\therefore$ $x = 13$

Hence, the speed of the boat in still water = 13 km/h

Chapter-6 : Algebraic Expressions

Exercise-1

1. Write the terms and their numerical coefficients for the following :

(a) $-5xy + 3y^2z$

Numerical cofficients of $-5xy, 3y^2z$ are $-5, 3$.

(b) $12p - 9q + 11$

Numerical cofficients of $12pm, -pq$ are 12, –9.

(c) $\frac{5}{3}xy - \frac{3}{2}y^2$

Numberical cofficients of $\frac{5}{3}xy, \frac{-3}{2}y^2$ are $\frac{5}{3}, \frac{-3}{2}$.

(d) $m^2 - m - 1$

Numerical cofficients of $m^2, -m,$ are 1, –1.

2. Classify the following as monomial, binomial and trinomial :

(a) $12x^2 + 8x + 4$ (Trinomial)

(b) $4ab^2c$ (Monomial)

(c) $-2x + 3$ (Binomial)

(d) $84a^2 - 3b^2 - 11a^2$ (Trinomial)

(e) $-15x + 6y$ (Binomial)

(f) $9m^2 + 4x^2 - 12m$ (Trinomial)

(g) $4a^2b - 8ab^2c + 12abc$ (Trinomial)

3. Add the following :

(a) $6x + 4y - 3z, 7x - 11y - 9z$

$(6x + 4y - 3z) + (7x - 11y - 9z)$

$= (6x + 7x) + (4y - 11y) + (-3z - 9z)$

$= 13x - 7y - 12z$

(b) $3ax - 9ax^2 + 1, 5 + 9ax^2 - 4ax$

$(3ax - 9ax^2 + 1) + (5 + 9ax^2 - 4ax)$

$= (3ax - 4ax) + (-9ax^2 + 9ax^2) + (1 + 5)$

$= -ax + 6$

(c) $15m^2n - 17mn + 8mn^2, 13m^2 - 15mn - 9mn^2, -12m^2n + 21mn + 8mn^2 - 14mn^2$

$= (15m^2n - 17mn + 8mn^2) + (13m^2n - 15mn - 9mn^2) + (-12m^2n + 21mn + 8mn^2 - 14mn^2)$

$= (15m^2n + 13m^2n - 12m^2n) + (-17mn - 15mn + 21mn) + (8mn^2 - 9mn^2 + 8mn^2 - 14mn^2)$

$= (28m^2n - 12m^2n) + (-32mn + 21mn) + (16mn^2 - 23mn^2)$

$= 16m^2n - 11mn - 7mn^2$

$= 16m^2n - 7mn^2 - 11mn$

(d) $5a^2 - 3b^2, 16b^2 - 3a^2, -11a^2 - 9b^2$

$= (5a^2 - 3b^2) + (16b^2 - 3a^2) + (-11a^2 - 9b^2)$

$= (5a^2 - 3a^2 - 11a^2) + (-3b^2 + 16b^2 - 9b^2)$

$= (5a^2 - 14a^2) + (16b^2 - 12b^2)$

$= -9a^2 + 4b^2$

(e) $\frac{5}{2}x^2 - \frac{3}{5}y, \frac{-11}{4}x^2 - \frac{13}{10}y, -\frac{1}{2}y + \frac{3}{4}x^2$

$$= \left(\frac{5}{2}x^2 - \frac{3}{5}y\right) + \left(\frac{-11}{4}x^2 - \frac{13}{10}y\right) + \left(\frac{-1}{2}y + \frac{3}{4}x^2\right)$$

$$= \left(\frac{5}{2}x^2 - \frac{11}{4}x^2 + \frac{3}{4}x^2\right) + \left(\frac{-3}{5}y - \frac{13}{10}y - \frac{1}{2}y\right)$$

$$= \left(\frac{5}{2} - \frac{11}{4} + \frac{3}{4}\right)x^2 + \left(\frac{-3}{5} - \frac{13}{10} - \frac{1}{2}\right)y$$

$$= \frac{(10 - 11 + 3)x^2}{4} + \left(\frac{-6 - 13 - 5}{10}\right)y$$

$$= \frac{2}{4}x^2 + \left(\frac{-24}{10}\right)y$$

$$= \frac{1}{2}x^2 - \frac{12}{5}y$$

(f) $ab - bc, bc - ca, ca - ab$

$= (ab - bc) + (bc - ca) + (ca - ab)$

$= (ab - ab) + (-bc + bc) + (-ca + ca)$

$= 0$

(g) $7p - 8q + 11r + 13, 10p - 13q + 18, -8p - 5q - 14r + 12, 5p + 17q + 14r$

$= (7p - 8q + 11r + 13) + (10p - 13q + 18) + (-8p - 5q - 14r + 12), + (5p + 17q + 14r)$

$= (7p + 10p - 8p + 5p) + (-8q - 13q - 5q + 17q) + (11r - 14r + 14r) + (13 + 18 + 12)$

$= (22p - 8p) + (-26q + 17q) + (25r - 14r) + (31)$

$= 14p - 9q + 11r + 43$

4. Subtract the following :

(a) $21ax + 15by - 6cz$ from $12ax + 6by + 11cz$

$= (12ax + 6by + 11cz) - (21ax + 15by - 6cz)$

$= 12ax + 6by + 11cz - 21ax - 15by + 6cz$

$= -9x - 9by + 17cz$

(b) $15 - 9p + 6r - 4q$ from $7p + 11q - 2r + 9$

$= (7p + 11q - 2r + 9) - (15 - 8p + 6r - 9q)$

$= 7p + 11q - 2r + 9 - 15 + 8p - 6r + 4q$

$= (7p + 8p) + (11q + 4q) - (2r - 6r) + (9 - 15 + 49)$

$= 15p + 15q - 8r - 6$

(c) $7a^2 - 3ab$ from $15a^2 - 6ab$

$= (15a^2 - 6ab) - (7a^2 - 3ab)$

$= 15a^2 - 6ab - 7a^2 + 3ab$

$= 8a^2 - 3ab$

(d) $a^2 - b^2$ from $9a^2 + 4b^2 - 8ab$

$= (9a^2 + 4b^2 - 8ab) - (a^2 - b^2)$

$= 9a^2 + 4b^2 - 8ab - a^2 + b^2$

$= 8a^2 + 5b^2 - 8ab$

5. What should be subtracted from $x^2 + y^2 + 2xy$ to get $x^2 + y^2$?

Let A be subtracted : $x^2 + y^2 + 2xy - A = x^2 + y^2$

or $A = (x^2 + y^2 + 2xy) - (x^2 + y^2)$

$A = x^2 + y^2 + 2xy - x^2 - y^2$

$A = 2xy$

$\therefore$ $= 2xy$ should be substrated

6. Subtract the sum of $12p - 3q + 6p^2q$ and $45q - 3p$ from the sum of $-3p + 7qp^2 + 11q$ and $-17p + 13q$.

Sum of first two expressions

$= (12p - 3q + 6p^2q) + (45q - 3p)$

$= 12p - 3q + 6p^2q + 45q - 3p$

$= 9p + 42q + 6p^2q$ ...(i)

Now, sum of the last two expressions

$= (-3p + 7qp^2 + 11q) + (-17p + 13q)$

$= -3p + 7qp^2 + 11q - 17p + 13q$

$= (-3p - 17p) + 7qp^2 + (11q + 13q)$

$= -20p + 7qp^2 + 24q$ ...(iii)

Now, we subtract (ii) from (i)

$= (-20p + 7qp^2 + 24q) - (9p + 42q + 6qp^2)$

$= -20p + 7qp^2 + 24q - 9p - 42q - 6qp^2$

$= -29p + qp^2 - 18q$

7. The perimeter of a triangle is $14a - 3b + 12c$. If the two sides of the triangle are $4a + 6b - 8c$ and $3a - 5b + 11c$, find the third side.

Perimeter of a triangle $= 14a - 3b + 12c$

Two sides of the triangle $= 4a + 6b - 8c$, and $3a - 5b + 11c$

Perimeter of a triangle = Sum of three sides

$14a - 3b + 12c = (4a + 6b - 8c) + (3a - 5b + 11c) +$ third side

$(14a - 3b + 12c) - (4a + 6b - 8c) - (3a - 5b + 11c) =$ third side

Third side $= (14a - 4a - 3a) - (3b + 6b - 5b) + (12c + 8c - 11c)$

$= 7a - 4b + 9c$

Exercise-2

1. Multiply the following :

(a) $-5x^2y \times 5y^2x^2$

$$= (-5 \times 5) \times (x^2y \times y^2x^2)$$
$$= -25x^4y^3$$

(b) $\frac{1}{2}abc \times (-4)ab^2$ $= \frac{1}{2} \times (-4) \times (abc \times ab^2)$

$$= \frac{-4}{2} \times a^2b^3c$$
$$= -2a^2b^3c$$

(c) $\frac{1}{7}p^3q^2 \times \left(\frac{-14}{5}\right)pqr \times \frac{5}{3}qr^2$

$$= \frac{1}{7} \times \left(\frac{-14}{5}\right) \times \frac{5}{3} \times (p^3q^2 \times pqr \times qr^2)$$
$$= \frac{-2}{3}p^4q^4r^3$$

(d) $\frac{11}{4}a^2b \times \frac{16}{12}ab^2 \times \left(\frac{-36}{22}\right)a^2b^2$

$$= \frac{11}{4} \times \frac{16}{12} \times \left(\frac{-36}{22}\right) \times \left(a^2b \times ab^2 \times a^2b^2\right)$$
$$= (-6) \times (a^5b^5)$$
$$= -6a^5b^5$$

2. Find the area of the rectangles with the following dimensions :

(a) $12xy, 2x^2y^2$

Area of rectangle $= l \times b$

$$= (12xy) \times (2x^2y^2)$$
$$= 24x^3y^3$$

(b) $5xy, 11x^3y^2$

Area of rectangle $= l \times b$

$$= 5xy \times 11x^3y^2$$
$$= 55x^4y^3$$

(c) $\frac{1}{4}xy, 15x^2y^2$

Area of rectangle $= l \times b$

$$= \frac{1}{4}xy \times 15x^2y^2$$
$$= \frac{15}{4}x^3y^3$$

3. Multiply the following binomials :

(a) $(11p - 9q)\,(4p + 17q)$

$= 11p \times (4p + 17q) - 9q\ (4p + 17q)$
$= 44p^2 + 187pq - 36pq - 153q^2$
$= 44p^2 + 151pq - 153q^2$

(b) $(5a + 11b)\ (8a + 9b)$
$5a \times (8a + 9b) + 11b \times (8a + 9b)$
$= (5a \times 8a) + (5a \times 9b) + (11b \times 8a) + (11b \times 9b)$
$= 40a^2 + 45ab + 88ab + 99b^2$
$= 40a^2 + 99b^2 + 133ab$

(c) $(3p - 2q)\ (p + q)$
$= 3p \times (p + q) - 2q \times (p + q)$
$= (3p \times p) + (3p \times q) - (2q \times p) - (2q \times q)$
$= 3p^2 + 3pq - 2pq - 2q^2$
$= 3p^2 - 2q^2 + pq$

(d) $(13xy - 3z)\ (5xy - 8z)$
$= 13xy\ (5xy - 8z) - 3z\ (5xy - 8z)$
$= (13xy \times 5xy) - (13xy \times 8z) - (3z \times 5xy) - (3z \times 8z)$
$= 65x^2y^2 - 104xyz - 15xyz + 24z^2$
$= 65x^2y^2 - 119xyz + 24z^2$
$= 65x^2y^2 + 24z^2 - 119xyz$

(e) $(2 \cdot 5a + 0 \cdot 3b)\ (1 \cdot 5c + 0 \cdot 7d)$
$= 2 \cdot 5a \times (1 \cdot 5c + 0 \cdot 7d) + 0 \cdot 3b \times (1 \cdot 5c + 0 \cdot 7d)$
$= (2 \cdot 5a \times 1 \cdot 5c) + (2 \cdot 5a \times 0 \cdot 7d) + (0 \cdot 3b \times 1 \cdot 5c) + (0 \cdot 3b \times 0 \cdot 7d)$
$= 3 \cdot 75ac + 1 \cdot 75ad + 0 \cdot 45bc + 0 \cdot 21bd$

(f) $\left(\frac{3}{2}z - \frac{1}{3}y\right)(6z - 12y)$

$= \frac{3}{2}z \times (6z - 12y) - \frac{1}{3}y\ (6z - 12y)$
$= \left(\frac{3}{2}z \times 6z\right) - \left(\frac{3}{2}z \times 12y\right) - \left(\frac{1}{3}y \times 6z\right) + \left(\frac{1}{3}y \times 12y\right)$
$= 9z^2 - 18zy - 2yz + 4y^2$
$= 9z^2 + 4y^2 - 20zy$

(g) $\left(5a^2 - \frac{1}{5}b^2\right)(10x + 5y)$

$= 5a^2 \times (10x + 5y) - \frac{1}{5}b^2\ (10x + 5y)$
$= (5a^2 \times 10x) + (5a^2 \times y) - \left(\frac{1}{5}b^2 \times 10x\right) - \left(\frac{1}{5}b^2 \times 5y\right)$
$= 50a^2x + 25a^2y - 2b^2x - b^2y$

4. Find the product :
(a) $(xyz) \times (6x^3 + 7x^2 - 12x + 21)$
$= (xyz \times 6x^3) + (xyz \times 7x^2) - (xyz \times 12x) + (xyz \times 21)$

$= 6x^4yz + 7x^3yz - 12x^2yz + 21xyz$

(b) $(4x^2 + 11) \times (3x^2 + 17x - 16)$

$= 4x^2 \times (3x^2 + 17x - 16) + 11 \times (3x^2 + 17x - 16)$

$= 12x^4 + 68x^3 - 64x^2 + 33x^2 + 187x - 176$

$= 12x^4 + 68x^3 - 31x^2 + 187x - 176$

(c) $2m^2n^2 \times (3m^2n^2 - 7mn + 10mn^2 - 18)$

$= (2m^2n^2 \times 3m^2n^2) - (2m^2n^2 \times 7mn) + (2m^2n^2 \times 10mn^2) - (2m^2n^2 \times 18)$

$= 6m^4n^4 - 14m^3n^3 + 20m^3n^4 - 36m^2n^2$

(d) $(7x - 4y) \times (6y^2x - 7x^2y + 2xy)$

$= 7x \times (6y^2x - 72x^2y + 2xy) - 4y \times (6y^2x - 7x^2y + 2xy)$

$= 42x^2y^2 - 49x^3y + 14x^2y - 24y^3x + 28x^2y^2 - 8xy^2$

$= 70x^2y^2 - 49x^3y - 24y^3x + 14x^2y - 8xy^2$

(e) $\left(a + \frac{1}{a}\right)\left(a^3 + \frac{1}{a^3}\right)$

$= (a \times a^3) + \left(a \times \frac{1}{a^3}\right) + \left(\frac{1}{a} \times a^3\right) + \left(\frac{1}{a} \times \frac{1}{a^3}\right)$

$= a^4 + \frac{1}{a^2} + a^2 + \frac{1}{a^4}$

5. Simplify : $2x^2(xy - 4) + 3y(x + 2)$ and find its value for

$2x^2(xy - 4) + 3y(x + 2)$

$= (2x^2 \times xy) - (2x^2 \times 4) + (3y \times x) + (3y + 2)$

$= 2x^3y - 8x^2 + 3xy + 6y$ …(i)

(a) Putting $x = 2,\ y = 3$ in eqn. (i)

$2x^3y - 8x^2 + 3xy + 6y$

$= 2 \times (2)^3 \times 3 - 8 \times (2)^2 + 3 \times 2 \times 3 + 6 \times 3$

$= 2 \times 2 \times 2 \times 2 \times 3 - 8 \times 2 \times 2 + 3 \times 2 \times 3 + 6 \times 3$

$= 48 - 32 + 18 + 18$

$= 16 + 18 + 18$

$= 52$

(b) $x = 1,\ y = 5$

Putting the value in eqn. (i)

$2xy^3 - 8x^2 + 3xy + 6y$

$= 2 \times 1 \times 1 \times 1 \times 5 - 8 \times 1 \times 1 + 3 \times 1 \times 5 + 6 \times 5$

$= 10 - 8 + 15 + 30$

$= 47$

6. Find the value of $(-5x^2) \times \left(\frac{-1}{15}xy^2\right) \times (-21x^2y^3)$, when $x = -1, x = 2$

$(-5x^2) \times \left(\frac{-1}{15}xy^2\right) \times (-21x^2y^3)$

$= -5 \times \frac{-1}{15} \times (-21) \times (x^2 \times xy^2 \times x^2y^3)$

$$= \frac{1}{3} \times (-21) \times (x^5 y^5)$$
$$= -7 \times x^5 y^5$$

Now putting the values $x = -1,\ y = 2$

$$= -7 \times (-1)^5 \times (2)^5$$
$$= -7 \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= -7 \times (-1) \times 32$$
$$= 7 \times 32 = 224$$

7. Simplify the following :

(a) $x^2 (x + 2y) - y(x^2 - y)$

$$= (x^2 \times x) + (x^2 \times 2y) - y \times x^2 + y \times y$$
$$= x^3 + 2x^2 y - x^2 y + y^2$$
$$= x^3 + x^2 y + y^2$$
$$= x^3 + y^2 + x^2 y$$

(b) $2a (2x + 3y) - 4x (a + 5y)$

$$= 2a \times 2x + 2a \times 3y - 4x \times a - 4x \times 5y$$
$$= 4ax + 6ay - 4ax - 20xy$$
$$= 6ay - 20xy$$

(c) $4 (2x + 5y + 8) + 3 (5x + 7y - 9)$

$$= 4 \times 2x + 4 \times 5y + 4 \times 8 + 3 \times 5x + 3 \times 7y + 3 \times (-9)$$
$$= 8x + 20y + 32 + 15x + 21y - 27$$
$$= 23x + 41y + 15$$

(d) $3pq (p - 2q) - 7p^2 q (q + 2p) - 9pq^2 (8 - 3p)$

$$= 3pq \times p + 3pq \times (-2q) - 7p^2 q \times q - 9p^2 q \times 2p$$
$$-9pq^2 \times 8 - 9pq^2 - 3p$$
$$= -3p^2 q - 6pq^2 - 7p^2 q^2 - 14p^3 q - 72pq^2 + 27p^2 q^2$$
$$= 20p^2 q^2 - 78pq^2 + 3p^2 q - 14p^3 q$$

(e) $l (l + m + n) - m (l - m + n) + n (-1 + m + n)$

$$= (l \times l + l \times m + l \times n) - (m \times l - m \times m + m \times n) + (n \times -l + n \times m + n \times n)$$
$$= l^2 + lm + ln - ml + m^2 - mn - nl + nm + n^2$$
$$= l^2 + m^2 + n^2 + (lm - lm) + (ln - ln) - mn + mn$$
$$= l^2 + m^2 + n^2$$

Exercise-3

1. Using suitable identities, find the following products :

(a) $(4x + 9y) (4x + 9y)$

$$= (4x + 9y)^2$$

Using the identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= (4x + 9y)^2 = (4x)^2 + 2 \times 4x \times 9y + (19y)^2$$
$$= 16x^2 + 72xy + 81y^2$$

(b) $(y+4)(y-4)$

Using the identity $(a+b)(a-b)=a^2-b^2$

$$(y+4)(y-4)=(y)^2-(4)^2$$
$$=y^2-16$$

(c) $(y-10)(y-10)$

$$=(y-10)^2$$

Using the identity $(a-b^2)=a^2-2ab+b^2$

$$=(y-10)^2=y^2-2\times y\times 10+(10)^2$$
$$=y^2-20y+100$$

(d) $(3x+4)(3x+5)$

Using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(3x+4)(3x+5)=(3x)^2+(4+5)\times 3x+4\times 5$

$$=9x^2+9\times 3x+20$$
$$=9x^2+27x+20$$

(e) $(x^2-ay)(x^2-ay)$

$$=(x^2-ay)^2$$

Now, using identity $(a-b)^2=a^2-2ab+b^2$

$$=(x^2)^2-2\times x^2\times ay+(ay)^2$$
$$=x^4-2x^2ay+a^2y^2$$

(f) $(-3p+7q)(-3p+7q)$

$$=(-3p+7q)^2$$

Now, using identity $(a-b)^2=a^2-2ab+b^2$

$$=(-3p+7q)^2=(-3p)^2-2\times(-3p)\times(7q)+(7q)^2$$
$$=9p^2+42pq+49q^2$$

(g) $\left(\dfrac{3x^2}{4}+\dfrac{4y^2}{7}\right)\left(\dfrac{3x^2}{4}-\dfrac{4y^2}{7}\right)$

Using identity $(a+b)(a-b)=a^2-b^2$

$$=\left(\frac{3x^2}{4}\right)^2-\left(\frac{4y^2}{7}\right)^2$$
$$=\frac{9x^4}{16}-\frac{16y^4}{49}$$

(h) $(0\cdot 5x+0\cdot 9y)(0\cdot 5x-0\cdot 9y)$

Using identity $(a+b)(a-b)=a^2-b^2$

$$=(0\cdot 5x)^2-(0\cdot 9y)^2=0\cdot 25x^2-0\cdot 81y^2$$

(i) $\left(\dfrac{7a}{2}+\dfrac{3b}{5}\right)\left(\dfrac{7a}{2}+\dfrac{3b}{5}\right)$

$$=\left(\frac{7a}{2}+\frac{3b}{5}\right)^2$$

Now, using identlity $(a+b)^2 = a^2 + b^2 + 2ab$

$$= \left(\frac{7a}{2} + \frac{3b}{5}\right)^2 = \left(\frac{7a}{2}\right)^2 + 2 \times \left(\frac{7a}{2}\right) \times \left(\frac{3b}{5}\right) + \left(\frac{3b}{5}\right)^2$$

$$= \frac{49a^2}{4} + \frac{21ab}{5} + \frac{9b^2}{25}$$

(j) $625 - 49x^2$

$$= (25)^2 - (7x)^2$$

Now, using identity $a^2 - b^2 = (a+b)(a-b)$

$$= (25 + 7x)(25 - 7x)$$

2. Simplify the following :

(a) $(ax+b)^2 - (ax-b)^2$

$$= [(ax)^2 + b^2 + 2ax \times b] - [(ax)^2 + b^2 - 2axb]$$

$$= a^2x^2 + b^2 + 2abx - a^2x^2 - b^2 + 2abx$$

$$= 2abx + 2abx$$

$$= 4abx$$

(b) $(abc+7)^2 - 14abc$

$(abc)^2 + 2abc \times 7 + (7)^2 - 14abc$

$$= a^2b^2c^2 + 14abc + 49 - 14abc$$

$$= a^2b^2c^2 + 49$$

(c) $(3 \cdot 5a - 4 \cdot 5b)^2 - (3 \cdot 5a + 4 \cdot 5b)^2$

$$= [(3 \cdot 5a)^2 + (4 \cdot 5b)^2 - 2 \times (3 \cdot 5a) \times (4 \cdot 5b)]$$

$$- [(3 \cdot 5a)^2 + (4 \cdot 5b)^2 + 2 \times (3 \cdot 5a) \times (4 \cdot 5b)]$$

$$= 12 \cdot 25a^2 + 20 \cdot 25b^2 - 31 \cdot 5ab - 12 \cdot 25a^2 - 20 \cdot 25b^2 - 31 \cdot 5ab$$

$$= -63 \cdot 0ab$$

(d) $(7x+9y)^2 + (9x+7y)^2$

$$= [(7x)^2 + 2 \times 7x + 9y + (9y)^2)] + [(9x)^2 + (2 \times 9x \times 7y) + (7y)^2]$$

$$= 49x^2 + 126xy + 81y^2 + 81x^2 + 126xy + 49y^2$$

$$= 49x^2 + 81y^2 + 81x^2 + 49y^2 + 252xy$$

$$= (49+81)x^2 + (81+49)y^2 + 252xy$$

$$= 130x^2 + 130y^2 + 252xy$$

3. Evaluate the following using suitable identities :

(a) $(92)^2$

$$= (90+2)^2$$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$

$$(90+2)^2 = (90)^2 + 2 \times 90 \times 2 + (2)^2$$

$$= 8464$$

(b) $(107)^2$

$$= (100+7)^2$$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$

$$(100+7)^2 = (100)^2 + 2 \times 100 \times 7 + (7)^2$$
$$= 10000 + 1400 + 49$$
$$= 11449$$

(c) $(5 \cdot 1)^2$

$$= (5 + 0 \cdot 1)^2$$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$

$$(5 + 0 \cdot 1)^2 = (5)^2 + 2 \times 5 \times (0 \cdot 1) + (0 \cdot 1)^2$$
$$= 25 + 1 + 0 \cdot 01$$
$$= 26 \cdot 01$$

(d) $(997)^2$

$$(1000 - 3)^2$$

Using identity $(a-b)^2 = a^2 - 2ab + b^2$

$$(1000 - 3)^2 = (1000)^2 - 2 \times 1000 \times 3 + (3)^2$$
$$= 1000000 - 6000 + 9$$
$$= 994009$$

(e) 103×97

$$= (100+3)\,(100-3)$$

Using identity $(a+b)\,(a-b) = a^2 - b^2$

$$(100+3)\,(100-3) = (100)^2 - (3)^2$$
$$= 10000 - 9$$
$$= 9991$$

(f) (194×206)

$$= (200-6) \times (200+6)$$

Using identity $(a+b)\,(a-b) = a^2 - b^2$

$$(200-6) \times (200+6) = (200)^2 - (6)^2$$
$$= 40000 - 36$$
$$= 39{,}964$$

(g) $75^2 - 25^2$

Using identity $a^2 - b^2 = (a+b)\,(a-b)$

$$(75)^2 - (25)^2 = (75+25)\,(75-25)$$
$$= (100) \times (50)$$
$$= 5000$$

(h) $7 \cdot 5 \times 7 \cdot 7$

$$= (7 + 0 \cdot 5)\,(7 + 0 \cdot 7)$$
$$= (7)^2 + 7 \times (0 \cdot 5 + 0 \cdot 7) + 0 \cdot 5 \times 0 \cdot 7$$
$$= 49 + 7 \times 1 \cdot 2 + 0 \cdot 35$$
$$= 49 + 8 \cdot 4 + 0 \cdot 35$$
$$= 57 \cdot 75$$

4. Given $x + y = 5$ and $xy = 36$, find the value of $x^2 + y^2$.

$$x + y = 5$$
$$xy = 36$$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$

$$(x+y)^2 = x^2 + 2xy + y^2$$

or $(x+y)^2 = x^2 + y^2 + 2xy$

$$(5)^2 = x^2 + y^2 + 2 \times 36 \qquad [\because \text{Given } x + y = 5, xy = 36]$$
$$25 = x^2 + y^2 + 72$$
$$x^2 + y^2 = 25 - 72$$

$\therefore$ $x^2 + y^2 = -47$

5. Given $2x + 3y = 14$ and $xy = 8$, find the value of $4x^2 + 9y^2$.

$$(2x + 3y) = 14$$

Using identity, $(a+b)^2 = a^2 + 2ab + b^2$

$$(2x+3y)^2 = 4x^2 + 12xy + 9y^2$$
$$(2x+3y)^2 = 4x^2 + 9y^2 + 12xy$$
$$(14)^2 = 4x^2 + 9y^2 + 12 \times 8$$
$$196 = 4x^2 + 9y^2 + 96$$
$$4x^2 + 9y^2 = 196 - 96$$
$$4x^2 + 9y^2 = 100$$

6. Verify the following using identities :

(a) $(x+a)(x-a) + (a^2 - x^2) = 0$

L.H.S. $= (x+a)(x-a) + a^2 - x^2$

Using identity $(a+b)(a-b) = (a^2 - b^2)$

$$= x^2 - a^2 + a^2 - x^2$$
$$= 0$$

L.H.S. = R.H.S.

(b) $(l^2 + m^2)^2 - 4l^2m^2 = (l^2 = m^2)$

L.H.S. $= (l^2 + m^2)^2 - 4l^2m^2$

$$= l^4 + m^4 + 2l^2m^2 - 4l^2m^2$$
$$= l^4 + m^4 - 2l^2m^2$$

R.H.S. $= (l^2 - m^2)^2$

Using identity $(a-b)^2 = a^2 - 2ab + b^2$

$$(l^2 - m^2)^2 = l^4 - 2l^2m^2 + m^4$$
$$= l^4 + m^4 - 2l^2m^2$$

L.H.S. = R.H.S.

7. Verify teidentity $(a-b)^2 = a^2 - 2ab + b^2$ for $a = 9$ and $b = 4$.

L.H.S. $= (a-b)^2$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$= (9)^2 + (4)^2 - 2 \times 9 \times 4$ [Given $a = 9, b = 4$]

$= 81 + 16 - 72$

$= 25$

R.H.S. $= a^2 - 2ab + b^2$

$= (9)^2 - 2 \times 9 \times 4 + (4)^2$ [Givn $a = 9, b = 4$]

$= 81 - 72 + 16$

$= 25$

L.H.S. = R.H.S.

8. Verify the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ for $x = 11$, $a = 4$ and $b = -7$.

$(x + a)(x + b) = x^2 + (a + b)x + ab$

L.H.S. $= (x + a)(x + b)$

$= (11 + 4)(11 + (-7))$ [Given $x = 11, a = 4, b = -7$]

$= 15 \times (11 - 7)$

$= 15 \times 4$

$= 60$

R.H.S. $= x^2 + (a + b)x + ab$

$= (11)^2 + (4 - 7) \times 11 + 4 \times (-7)$ [Given $x = 11, a = 4, b = -7$]

$= 121 - (3 \times 11) - 28$

$= 121 - 33 - 28$

$= 121 - 61 = 60$

L.H.S. = R.H.S.

9. Find the value of $9x^2 - 12xy + 4y^2$ for $x = \frac{-1}{2}$ and $y = \frac{-2}{3}$.

$9x^2 - 12xy + 4y^2$

$= 9 \times \left(\frac{-1}{2}\right)^2 - 12 \times \left(\frac{-1}{2}\right) \times \left(\frac{-2}{3}\right) + 4 \times \left(\frac{-2}{3}\right)^2$

$= 9 \times \frac{1}{4} - 12 \times \frac{1}{2} \times \frac{2}{3} + 4 \times \frac{4}{9}$

$= \frac{9}{4} - \frac{24}{6} + \frac{16}{9}$

$= \frac{81 - 144 + 64}{36}$

$= \frac{145 - 144}{36} = \frac{1}{36}$

10. Find the value of $(89x - 5y)^2 + 1780xy$

$(89x - 5y)^2 + 1780xy$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$= (89x)^2 - 2 \times 89x \times 5y + (15y)^2 + 1780xy$

$= 7921x^2 - 890xy + 25y^2 + 1780xy$

$= 7921x^2 + 25y^2 + 890xy$

Chapter-7 : Factorisation

Exercise-1

1. Factorise the following expressions :

(a) $36y^2 + 12y + 1$

$= (6y)^2 + 2 \times 6 \times 1y + (1)^2$

$= (6y+1)^2$ $\quad [\because \; a^2 + 2ab + b = (a+b)^2]$

$= (6y+1)\,(6y+1)$

(b) $(x+1)^2 - 4x$

$= (x^2 + 2 \times x \times 1 + 1) - 4x$

$= x^2 + 2x + 1 - 4x$

$= x^2 - 2x + 1$

$= (x)^2 - 2 \times 1 \times x + (1)^2$ $\quad [\because \; a^2 - 2ab + b^2 = (a-b)^2]$

$= (x-1)^2$

$= (x-1)\,(x-1)$

(c) $9x^2y^2 + 6xyz + z^2$

$= (3xy)^2 + 2 \times 3xy \times z + (z)^2$

$= (3xy + z)^2$ $\quad [\because \; a^2 - 2ab + b^2 = (a-b)^2]$

$= (3xy + z)\,(3xy + z)$

(d) $x - y - x^2 + y^2$

$= (x - y) - (x^2 - y^2)$

$= (x - y) - (x - y)\,(x + y)$

$= (x - y)\,[1 - (x + y)]$

(e) $9x^2 + 42x + 49 - y^4$

$= (3x)^2 + 2 \times 6 \times 7x + (7)^2 - (y)^2$

$= (3x + 7)^2 - (y)^2$ $\quad [\because \; a^2 + 2ab + b^2 = (a+b)^2]$

$= (3x + 7 + y)\,(3x + 7 - y)$ $\quad [\because \; a^2 - b^2 = (a+b)\,(a-b)]$

(f) $25\,(x + y)^2 - 16\,(x - y)^2$

$25\,(x^2 + 2xy + y^2) - 16\,(x^2 - 2xy + y^2)$

$= 25x^2 + 50xy + 25y^2 - 16x^2 - 32xy - 16y^2$

$= (25x^2 - 16x^2) + (50xy - 32xy) + (25y^2 - 16y^2)$

$= 9x^2 + 18xy + 9y^2$

$= (3x)^2 + 2 \times 3x \times 3y + (3y)^2$

$= (3x + 3y)^2$ $\quad [\because \; a^2 + ab + b^2 = (a+b)^2]$

$= (3x + 3y)\,(3x + 3y)$

(g) $25x^2 + 30x + 9$

$= (5x)^2 + 2 \times 5x \times 3 + (3)^2$

$= (5x + 3)^2$ $\quad [\because (a^2 + 2ab + b^2) = (a+b)^2 1]$

$= (5x + 3)\ (5x + 3)$

(h) $25x^2 - 75x^2 + 100x^4$

$= 25x^2\ (1 - 3x + 4x^2)$

(i) $19x^2 + 57x$

$= 19x\ (x + 3)$

(j) $36l^2m - 63lm^3$

$= 9lm\ (4l - 7m^2)$

(k) $24 + a^2b + 8b + 3a^2$

$= 24 + 8b + 3a^2 + a^2b$

$= 8\ (3 + b) + a^2\ (3 + b)$

$= (8 + a^2)\ (3 + b)$

2. Factorise the following using identities :

(a) $x^2 + 2 + \frac{1}{x^2}$

$= (x)^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$= \left(x + \frac{1}{x}\right)^2$ [$\because$ $(x^2 + 2ab + b^2) = (a + b)^2$]

$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

(b) $64a^2 - 25b^2$

$= (8a)^2 - (5b)^2$

$= (8a + 5b)\ (8a - 5b)$ [$\because$ $a^2 - b^2 = (a + b)\ (a - b)$]

(c) $169 - 25y^2$

$= (13)^2 - (5y)^2$

$= (13 + 5y)\ (13 - 5y)$ [$\because$ $a^2 - b^2 = (a + b)\ (a - b)$]

(d) $9y^2 - 24xy + 16x^2$

$= (3y)^2 - 2 \times 3y \times 4x + (4x)^2$

$= (3y - 4x)^2$ [$\because$ $a^2 - 2ab + b^2 = (a - b)^2$]

$= (3y - 4x)\ (3y - 4x)$

(e) $x^2 + 12x + 36$

$= (x)^2 + 2 \times x \times 6 + (6)^2$

$= (x + 6)^2$ [$\because$ $(a^2 + 2ab + b^2) = (a + b)^2$]

$= (x + 6)\ (x + 6)$

(f) $16m^2 + 8mn + n^2$

$= (4m)^2 + 2 \times 4m \times n + (n)^2$

$= (4m + n)^2$ [$\because$ $a^2 + 2ab + b^2 = (a + b)^2$]

$= (4m + n)\ (4m + n)$

(g) $a^4 - 16(b-c)^4$

$= (a^2)^2 - [4(b-c)^2]^2$

$= [a^2 + 4(b-c)^2][a^2 - 4(b-c)^2]$

$= [a^2 + \{2(b-c)\}^2][a^2 - \{2(b-c)\}^2]$

$= [a^2 + (2b-2c)^2][\{a + 2(b-c)\}][(a - 2(b-c)]$

$= [a^2 + (ab-2c)^2][a + 2b - 2c][a - 2b + 2c]$

(h) $49a^2 - b^2$

$= (7a)^2 - (b)^2$

$= (7a+b)(7a-b)$ $[\because \ a^2 - b^2 = (a+b)(a-b)]$

3. Factorise :

(a) $x^4 - 1$

$= (x^2)^2 - (1)^2$

$= (x^2+1)(x^2-1)$ $[\because \ a^2 - b^2 = (a+b)(a-b)]$

$= (x^2+1)(x+1)(x-1)$ [Again using the same fomula]

(b) $x^4 - 16$

$= (x^2)^2 - (2^2)^2$

$= (x^2+2^2)(x^2-(2)^2)$ $[\because \ a^2 - b^2 = (a+b)(a-b)]$

$= (x^2+4)(x^2-(2)^2)$

$= (x^2+4)(x+2)(x-2)$ [Again using the same formula]

Exercise-2

1. Factorise the following using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$:

(a) $x^2 - 11x - 42$

The constant term 42 can be factorised as

$-42 = -14 \times 3$ and sum $-14 + 3 = -11$

$x^2 - 11x - 42 = x^2 + (-14+3)x - 14 \times 3$

$= x^2 - 14x + 3x - 42$

$= x(x-14) + 3(x-14)$

$= (x+3)(x-14)$

(b) $a^2 + 14a + 48$

The constant term 48 can be factorised as

$48 = 6 \times 8$ and sum $6 + 8 = 14$

$= a^2 + (6+8)a + 48$

$= a^2 + 6a + 8a + 48$

$= a(a+6) + 8(a+6)$

$= (a+8)(a+6)$

(c) $x^2 - 9x + 20$

The constant term 20 can be factorised as

$20 = (-5) \times (-4)$ whose sum is $-5 - 4 = -9$

$$x^2 - 9x + 20 = x^2 + (-5 - 4)x + 20$$
$$= x^2 - 5x - 4x + 20$$
$$= x(x - 5) - 4(x - 5)$$
$$= (x - 4)(x - 5)$$

(d) $x^2 - 6x + 5$

The constant term 5 can be factorised $5 = (-5) \times (-1)$ and sum $-5 - 1 = -6$

$$x^2 - 6x + 5 = x^2 + (-5 - 1)x + 5$$
$$= x^2 - 5x - x - 5$$
$$= x(x - 5) - 1(x - 5)$$
$$= (x - 1)(x - 5)$$

(e) $y^2 - 22y + 121$

The constant term 121 can be factorised as $121 = (-11) \times (-11)$ and sum $-11 - 11 = -22$

$$y^2 - 22y + 121 = y^2 + (-11 - 11)y + 121$$
$$= y^2 - 11y - 11y - 121$$
$$= y(y - 11) - 11(y - 11)$$
$$= (y - 11)(y - 11)$$

(f) $x^2 - 13x + 36$

The constant term 36 can be factorised as $36 = -9 \times -4$, whose sum is -13

$$x^2 - 13x + 36 = x^2 + (-9 - 4)x + 36$$
$$= x^2 - 9x - 4x + 36$$
$$= x(x - 9) - 4(x - 9)$$
$$= (x - 9)(x - 4)$$

(g) $x^2 + 3x + 2$

The constant term 2 can be factorised $2 = 2 \times 1$, whose sum is 3.

$$x^2 + 3x + 2 = x^2 + (2 + 1)x + 2$$
$$= x^2 + 2x + 1x + 2$$
$$= x(x + 2) + 1(x + 2)$$
$$= (x + 2)(x + 1)$$

(h) $z^2 + 22x + 35$

Question is wrong.

2. Factorise :

(a) $x^2 + 3xy - 18y^2$

By splitting the middle term as $6 \times (-3) = -18$ and $6 + (-3) = 3$

$$x^2 + 3xy - 18y^2 = x^2 + 6xy - 3xy - 18y^2$$
$$= x(x + 6y) - 3y(x + 6y)$$
$$= (x + 6y)(x - 3y)$$

(b) $6a^2 - 13ab + 2b^2$

By splitting the middle term as $12 \times 1 = 12$ and $-12 - 1 = -13$

$6a^2 - 13ab + 2b^2$

$$= 6a^2 - 12ab - 1ab + 2b^2$$
$$= 6a(a - 2b) - b(a - 2b)$$

$= (a - 2b)(6a - b)$

(c) $6x^2 - 5xy - 6y^2$

By splitting the middle term as $4 \times (-9) = -36$ and $4 + (-9) = -5$

$$6x^2 - 5xy - 6y^2 = 6x^2 + 4xy - 9xy - 6y^2$$
$$= 2x(3x + 2y) - 3y(3x + 2y)$$
$$= (3x + 2y)(2x - 3y)$$

(d) $4x^2 + 12xy + 9y^2$

By splitting the middle term as $6 \times 6 = 36$ and $6 + 6 = 12$

$$4x^2 + 12xy + 9y^2 = 4x^2 + 6xy + 6xy + 9y^2$$
$$= 2x(2x + 3y) + 3y(2x + 3y)$$
$$= (2x + 3y)(2x + 3y)$$

(e) $4a^2 + 4a - 80$

$$= 4(a^2 + a - 20)$$
$$= 4(a^2 + 5a - 4a - 20) \qquad [\therefore\ 20 = 5 \times 4 \text{ and } 5 - 4 = 1]$$
$$= 4\{a(a + 5) - 4(a + 5)\}$$
$$= 4\{(a + 5)(a - 4)\}$$
$$= 4(a - 4)(a + 5)$$

(f) $3x^2 + 22x + 35$

By splitting the middle term as $7 \times 15 = 105$ and $7 + 15 = 22$

$$3x^2 + 22x + 35 = 3x^2 + 7x + 15x + 35$$
$$= x(3x + 7) + 5(3x + 7)$$
$$= (3x + 7)(x + 5)$$

(g) $2a^2 - 7a + 6$

By splitting the middle term as $-3 \times -4 = 12$ and $-3 - 4 = -7$

$$2a^2 - 7a + 6 = 2a^2 - 3a - 4a + 6$$
$$= a(2a - 3) - 2(2a - 3)$$
$$= (2a - 3)(a - 2)$$

Exercise-3

1. Simplify :

(a) $\dfrac{x^2 - 9}{5x - 15}$

$$\frac{x^2 - (3)^2}{5(x - 3)} = \frac{(x + 3)(x - 3)}{5(x - 3)}$$
$$= \frac{x + 3}{5}$$

(b) $\dfrac{3x + 12}{16 - x^2}$

$$= \frac{3(x + 4)}{(4)^2 - (x)^2} = \frac{3(x + y)}{(4 + x)(4 - x)}$$

$$= \frac{3}{(4 - x)}$$

(c) $\frac{16a^4 + 8a^3 + 12a}{4a}$

$$= \frac{4a\,(4a^3 + 2a^2 + 3)}{49a}$$

$$= 4a^3 + 2a^2 + 3$$

(d) $\frac{30a^2b^2c^2}{6abc}$

$$= 5abc$$

2. Divide by factorisation :

(a) $x^2 + 14x + 48$ by $x + 6$

Factorise the dividend

$$x^2 + 14x + 48 = x^2 + 6x + 8x + 48$$
$$= x\,(x + 6) + 8\,(x + 6)$$
$$= (x + 6)\,(x + 8)$$

Now $x^2 + 14x + 48 \div (x + 6)$

$$\frac{x^2 + 14x + 48}{x + 6} = \frac{(x + 6)\,(x + 8)}{(x + 6)}$$
$$= (x + 8)$$

(b) $x^2 - 14x - 51$ by $x + 3$

Factorise the dividend

$$x^2 - 14x - 51 = x^2 - 17x + 3x - 51$$
$$= x\,(x - 17) + 3\,(x - 17)$$
$$= (x - 17)\,(x + 3)$$

Now, $\frac{x^2 - 14x - 51}{(x + 3)} = \frac{(x - 17)\,(x + 3)}{(x + 3)}$

$$= (x - 17)$$

(c) $x^2 + 2x + 1$ by $x + 1$

Factorise the dividend

$$x^2 + 2x + 1 = x^2 + 1x + 1x + 1$$
$$= x(x + 1) + 1\,(x + 1)$$
$$= (x + 1)\,(x + 1)$$

Now, $\frac{x^2 + 2x + 1}{x + 1} = \frac{(x + 1)\,(x + 1)}{(x + 1)}$

$$= (x + 1)$$

(d) $4x^2 - 9$ by $2x - 3$

Factorise the dividend

$$4x^2 - 9 = (2x)^2 - (3)^2$$
$$= (2x + 3)\,(2x - 3)$$

$$\text{Now, } \frac{4x^2 - 9}{(2x - 3)} = \frac{(2x+3)(2x-3)}{(2x-3)}$$

$$= (2x + 3)$$

(e) $5x^2 + 10x + 5$ by $x + 1$

Factorise the dividend

$$5x^2 + 10x + 5 = 5x^2 + 5x + 5x + 5$$

$$= 5x(x+1) + 5(x+1)$$

$$= (x+1)(5x+5)$$

$$\text{Now, } \frac{5x^2 + 10x + 5}{x + 1} = \frac{(x+1)(5x+5)}{(x+1)}$$

$$= 5x + 5$$

$$= 5(x+1)$$

(f) $36a^2 + 12ab - 15b^2$ by $2a - b$

Factorise the dividend

$$36a^2 + 12ab - 15b^2$$

$$= 36a^2 + 30ab - 18ab - 15b^2$$

$$= 6a(6a + 5b) - 3b(6a + 5b)$$

$$= (6a + 5b)(6a - 3b)$$

$$\text{Now, } \frac{36a^2 + 12ab - 15b^2}{(2a - b)} = \frac{(6a+5b)(6a-3b)}{(2a-b)}$$

$$= \frac{(6a+5b)\,3\,(2a-b)}{(2a-b)}$$

$$= 3(6a + 5b)$$

3. Divide the following by the long division method :

(a) $12a^3 - 2a^2 + a + 1$ by $3a + 1$

$$\begin{array}{r|l}
 & 4a^2 - 2a + 1 \\
\hline
3a + 1 & 12a^3 - 2a^2 + a + 1 \\
 & 12a^3 - 4a^2 \\
 & - \quad\quad - \\
\hline
 & \quad - 6a^2 + a + 1 \\
 & \quad - 6a^2 - 2a \\
 & \quad + \quad\quad + \\
\hline
 & \quad\quad 3a + 1 \\
 & \quad\quad 3a + 1 \\
 & \quad\quad - \quad - \\
\hline
 & \quad\quad\quad 0 \\
\hline
\end{array}$$

So, the quotient $= 4a^2 - 2a + 1$ and remainder $= 0$

(b) $9 - 15y + 4y^2$ by $y - 3$

Here, dividend $= 4y^2 - 15y + 9$

$$
\begin{array}{r|l}
 & 4y - 3 \\
\hline
y - 3 & 4y^2 - 15y + 9 \\
 & 4y^2 - 12y \\
 & - \quad + \\
\hline
 & -3y + 9 \\
 & -3y + 9 \\
 & + \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

So the quotient $= 4y - 3$

and remainder $= 0$

(c) $x^2 + 12x + 38$ by $x + 7$

$$
\begin{array}{r|l}
 & x + 5 \\
\hline
x + 7 & x^2 + 12x + 38 \\
 & x^2 + 7x \\
 & - \quad - \\
\hline
 & 5x + 38 \\
 & 5x + 35 \\
 & - \quad - \\
\hline
 & 3 \\
\hline
\end{array}
$$

So, the quotient $= x + 5$

and remainder $= 3$

(d) $x^3 - 6x^2 + x + 8$ by $x + 1$

$$
\begin{array}{r|l}
 & x^2 - 7x + 8 \\
\hline
x + 1 & x^3 - 6x^2 + x + 8 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + x + 8 \\
 & -7x^2 - 7x \\
 & + \quad + \\
\hline
 & 8x + 8 \\
 & 8x + 8 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

So, the quotient $= x^2 - 7x + 8$

and remainder $= 0$

(e) $x^3 - 1$ by $x - 1$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x - 1 & x^3 - 1 \\
 & x^3 - x^2 \\
 & - \quad + \\
\hline
 & x^2 - 1 \\
 & x^2 - x \\
 & - \quad + \\
\hline
 & x - 1 \\
 & x - 1 \\
 & - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

So the quotient $= x^2 + x + 1$

and remainder $= 0$

(f) $12p + 10p^3 + 8p^4 + 15$ by $5 + 4p$

$8p^4 + 10p^3 + 12p + 15$ by $4p + 5$

$$\begin{array}{r|l} & 2p^3 + 3 \\ \hline 4p+5 & 8p^4 + 10p^3 + 12p + 15 \\ & 8p^4 + 10p^3 \\ & - \quad - \\ \hline & 12p + 15 \\ & 12p + 15 \\ & - \quad - \\ \hline & 0 \\ \hline \end{array}$$

So the quotient $= 2p^3 + 3$

and remainder $= 0$

4. Solve the following :

(a) $156x^4y^2z^3 \div (-4xyz)$

$$= \frac{156x^4y^2z^3}{-4xyz}$$

$$= -39x^3yz^2$$

(b) $16x^2y^2 + 12xy^2 - 8xy \div (-2xy)$

$$= \frac{16x^2y^2 + 12xy^2 - 8xy}{(-2xy)}$$

$$= \frac{16x^2y^2}{-2xy} - \frac{12xy^2}{2xy} + \frac{8xy}{2xy}$$

$$= -8xy - 6y + 4$$

(c) $16x^2y^2 + 12xy^2 - 8xy \div (-2xy)$

$$= \frac{16x^2y^2 + 12xy^2 - 8xy}{-2xy}$$

$$= \frac{16x^2y^2}{-2xy} + \frac{2xy^2}{-2xy} + \frac{8xy}{2xy}$$

$$= -8xy - 6y + 4$$

(d) $(2x^3 - x^2) \div (2x - 1)$

$$= \frac{(2x^3 - x^2)}{(2x - 1)}$$

$$= \frac{x^2(2x - 1)}{(2x - 1)}$$

$$= x^2$$

(e) $72x^2(4y^2 - 25) \div x(12y + 30)$

$$= \frac{72x^2(4y^2 - 25)}{x(12y + 30)}$$

$$= \frac{72x^2 \{(2y)^2 - (5)^2\}}{x \times 6\,(2y+5)}$$

$$= \frac{72x^2 \{(2y-5)\,(2y+5)\}}{6x\,(2y+5)}$$

$$= 12x\,(2y-5)$$

Chapter-8 : Playing with Numbers

Exercise-1

1. Without actually adding or dividing, find the quotient when the sum of :

(a) 49 and 94 is divided by 11 and 13

Their sum $= 49 + 94 = 143$

Dividing this by 11, we get quotient $\frac{143}{11} = 13$

and dividing this by 13, we get quotient 11

$$= \frac{143}{13} = 11$$

(b) 54 and 45 is divided by 11 and 9

Their sum $= 54 + 45 = 99$

Dividing this by 11, we get quotient

$$= \frac{99}{11} = 9$$

and dividing this by 9, we get quotient

$$= \frac{99}{9} = 11$$

2. The sum of the numbers 987, 879 and 798 is divided by 111, 24 and 37. Find the quotient in each case.

Sum of numbers $= 987 + 879 + 798 = 2664$

Dividing by 111, we get quotient

$$= \frac{2664}{111} = 24$$

Dividing by 24, we get quotient

$$= \frac{2664}{24} = 111$$

Now, dividing by 37, we get quotient

$$= \frac{2664}{37} = 72$$

3. The difference between 298 and 892 is divided by 6, 9 and 33. Find the quotient in each case.

Let $a = 8$, $b = 9$, $c = 2$

$$\text{Quotient} = \frac{\text{Dividend}}{\text{Divisor}}$$

$$= \frac{99 - (a - c)}{\text{divisor}} = \frac{99\,(8-2)}{6} = 99$$

$\therefore$ Quotient = 99

Now dividing by 9, we get

$$\text{Quotient} = \frac{99\,(a-c)}{9}$$
$$= 11\,(8-2)$$
$$= 11 \times 6$$

$\therefore$ Quotient = 66

and now dividing by 33, we get

$$\text{Quotient} = \frac{99\,(a-c)}{33}$$
$$= 3\,(8-2)$$

$\therefore$ Quotient $= 3 \times 6 \quad = 18$

4. Without actual division, find out the quotient when the sum of the number 167, 716 and 671 is divided by :

(a) 111

Their sum $= 167 + 716 + 671 = 1554$

Dividing this by 111, we get quotient $= \frac{1554}{111} = 14$

and dividing this by 14, we get quotient $= \frac{1554}{14} = 111$

(b) 37

Their sum $= 167 + 716 + 671 = 1554$

Dividing this by 37, we get quotient $= \frac{1554}{37} = 42$

and dividing this by 42, we get quotient $= \frac{1554}{42} = 37$

(c) 14

Their sum $= 167 + 716 + 671 = 1554$

Dividing this by 14, we get quotient $= \frac{1554}{14} = 111$

and dividing this by 111, we get quotient $= \frac{1554}{111} = 14$

Exercise-2

1. If $814y$ is divisible by 6, where y is a digit, find the values of y.

It is given that $814y$ is divisible by 6

$\therefore$ It is a multiple of both 2 and 3

$814y$ is a multiple of 3

$\therefore$ $8 + 1 + 4 + y$ is a multiple of 3

$13 + y = 0, 3, 6, 9, 12, 15$ ... (i)

$13 + y$ is a multiple of 3 and $y = 0, 1, 2, \ldots\ldots\ldots, 9$

i.e. $13 + y = 13, 14, 15, 16, 17, \ldots\ldots, 30$ ... (ii)

From equation (i) and (ii)

$13 + y = 15$ or $13 + y = 21$

$y = 15 - 13$ or $y = 21 - 13$

$y = 2$ $y = 8$

y can take place any value form 0 to 9 as ones digit is already even.

$\therefore$ $y = 2, 8$

2. Check the divisibility of 1809 by 9.

Adding the digits of the given number, we get

$1 + 8 + 0 + 9 = 18$ and $18 - 9 = 2$

$\therefore$ 1809 is divisible by 9.

3. Replace b by the smallest possible digit so that $123b$ is dividable by

(a) 2

$123b$ is a multiple of 2, where

$1 + 2 + 3 + b$ is a multiple of 2

$6 + b$ is a multiple of 2

i.e., $6 + b = 0, 2, 4, 6$

$6 + b = 6$

$b = 6 - 6$

$b = 0$

(b) 3

$123b$ is divisible by 3

the sum of the digits is a multiple of 3

$1 + 2 + 3 + b = 0, 3, 6$

$6 + b = 0, 3, 6, \ldots\ldots$ ... (i)

$6 + b = 6, 7, 8, 9, 10, \ldots\ldots, 15$ ... (ii)

[where $b = 0, 1, 2, \ldots\ldots, 9$] ... (ii)

From (i) and (ii)

$6 + b = 6$ or $6 + b = 6$

$b = 6 - 6 = 0$ $b = 6 - 6 = 0$

Hence $b = 0$

(c) 10

$123b$ is divisible by 10, it is also divisible by 2 and 5

$123b$ is divisible by 2.

b is an even digit

$b = 0, 2, 4, 6, 8$

$123b$ is multiple of 5

$b = 0$ or 5 [Number ending with 0 or 5 is divisible by 5]

(d) 9

Since the giveen number is divisible by 9, the sum of the digits must be a multiple of 9.

$1 + 2 + 3 + b = 0, 9, 18, 27, \ldots$

$6 + b = 0, 9, 18, 27, \ldots$ ... (i)

But b is a digit

$b = 0, 1, 2, 3, \ldots\ldots, 9$

$6 + b = 6, 7, 8, 9, 10, 11, 12, \ldots.$ (ii)

From equation (i) and (ii) it follows that

$6 + b = 9$

$b = 9 - 6 = 3$

4. If $312x$ is a multiple of 5, where x is a digit, what is the least value of x?

Solution : $312x$ is a multiple of 5

$x = 0$ [Number ending with 0 is divisible by 5]

5. If $376x5$ is divisible by 3, where x is a digit, what is the value of x?

Solution : Since the given number is divisible by 3, the sum of the digits must be a multiple of 3.

$$3 + 7 + 6 + x + 5 = 0, 3, 6, 9, 12, 15, 18, 21, 24, ... \quad ...(i)$$

$$21 + x = 0, 3, 6, 9, 12, 15, 18, 21, 24, ...$$

But x is a digit and can have values 0 to 9

$$x = 0, 1, 2, 3, ..., 9$$

$$21 + x = 21, 22, 23, 24, ..., 30 \quad ...(iii)$$

From equations (i) and (ii), it follows that

$$21 + x = 24$$

$$x = 24 - 21$$

$$x = 3$$

Therefore, x can be equal to 3.

Exercise-3

1. Find the values of the letters in the following cryptarithms giving reasons for the values chosen :

(a)

	3	A	5
+	6	2	B
	B	7	A

In hundreds column, $3 + 6 = B$ so that $B = 9$ is confirmed as C is not given for thousands place in the question. Now put $B = 9$ in ones column, $5 + B = A$

or $5 + 9 = A$ or $A = 14$

So that, we write $A = 4$ and 1 is carried forward to tens column, where $1 + a + 2 = 7$

$1 + 4 + 2 = 7$ which is correct.

Next, if $3 + 6 = B = 9$

Hence, $A = 4, B = 9$ is the correct solution.

(b)

	4	A
+	7	9
C	B	5

If we add A and 9, we get a number with 5 in the ones place. It means that $A = 6$. When we add A and 9, we get 15, so we carry forward the 1 to the next place, that is, tens place. Where $1 + 4 + 7 = B$ or $B = 12$. We carry forward the 1 to the next place, that is, hundreds place, So, $C = 1$.

Hence $A = 6, B = 2, C = 1$

(c)

$$\begin{array}{rcccc} & A & 8 & 4 & B \\ - & C & 8 & 1 & A \\ \hline & A & 0 & A & 5 \\ \hline \end{array}$$

If we subtract $4-1= A$, we get a number 3 in the tens place. It means that $A = 3$. When we put the value of $A5$ in ones place, $B - A = 5$, $B - 3 = 5$ we get a number 5 in the ones place. It means $B = 8$.

Next $A - C = A$, *i.e.* if we subtract from 3, we get 3. It means $C = 0$.

Hence, $A = 3$, $B = 8$, $C = 0$ is the correct solution.

(d)

$$\begin{array}{rccc} & 8 & A & 5 \\ + & Q & P & 8 \\ \hline & 1 & A & 3 \\ \hline \end{array}$$

If ones column, $5+8=13$, so we write 3 in unit place and carry the 1 forward to tens column, where $1+A+P=13$ or $1+7+5=13$, so we get $A = 7$, $P = 5$ and 1 is carried forward to hundreds column, where $1+8+Q = A$ or $1+8+8=17$, SO $Q = 8$, $A = 7$ and 1 is carried forward to next place.

Hence, $A = 7$, $P = 5$ and $Q = 8$ is the correct solution.

(f)

$$\begin{array}{rccc} & B & A & 9 \\ - & 3 & 1 & A \\ \hline & 5 & 5 & 3 \\ \hline \end{array}$$

If we subtract A from 9, we get a number 3 in the ones place. It means that $A = 6$.

Next, if 1 is subtracted from A, we get a number 5 in the tens place. It means that $A = 6$.

Next, if we subtract 3 from B, we get a number 5. It means that $B = 8$.

Hence, $A = 6$, $B = 8$ is the correct solution :

2. Find the values of letters in the following :

(a)

$$\begin{array}{ccc} & 3 & B \\ & \times & A \\ \hline 2 & 5 & 2 \\ \hline \end{array}$$

In the given puzzle, we have $B \times A = 2$

either $A = 7$, $B = 6$ or $A = 6$, $B = 7$

If $A = 7$, $B = 6$

the given puzzle reducess to

$6 \times 7 = 42$

$$\begin{array}{ccc} & 3 & 6 \\ & \times & 7 \\ \hline 2 & 5 & 2 \\ \hline \end{array} \qquad \text{4 is carried forward}$$

$\therefore$ $A = 7$, $B = 6$ is the required solution.

(b)

$$\begin{array}{cccc} & A & 5 & B \\ & & \times & B \\ \hline 2 & 7 & 3 & B \\ \hline \end{array}$$

In the given puzzle, we have $B \times B = B$

Let $B = 6$

Now, the given puzzle reduces to

$$\begin{array}{ccc} A & 5 & 6 \\ & \times & 6 \\ \hline 7 & 3 & 6 \\ \hline \end{array}$$

Now, $6 \times 6 = 36$
3 is carried forward
$A = 4$

Now, the given puzzle

$$\begin{array}{cccc} & 4 & 5 & 6 \\ & & \times & 6 \\ \hline 2 & 7 & 3 & 6 \\ \hline \end{array}$$

Now $4 \times 6 = 24$
2 is carried forward to the next place.

$\therefore$ $A = 4$, $B = 6$ is the required solution

(c)

$$\begin{array}{ccc} & 7 & A \\ & \times & 3 \\ \hline B & B & A \\ \hline \end{array}$$

Here, we have $A \times 3 = A$
Let $A = 5$ then $5 \times 3 = 15$
So, $A = 5$

Now, the given puzzle reduces to

$$\begin{array}{ccc} & 7 & 5 \\ & \times & 3 \\ \hline B & B & 5 \\ \hline \end{array}$$

Now $5 \times 3 = 15$ and 1 is carried forward and $7 \times 3 = 21$
$21 + 1 = 22$

Carried forward $B = 2$ be the next place.

$\therefore$ $A = 5$, $B = 2$ is the required solution.

$$\begin{array}{ccc} & 7 & 5 \\ & \times & 3 \\ \hline 2 & 2 & 5 \\ \hline \end{array}$$

(d)

$$\begin{array}{ccc} & A & 9 \\ & \times & B \\ \hline A & A & 1 \\ \hline \end{array}$$

In the given puzzle, we have $9 \times B = 1$
Now, taking $B = 9$ the given puzzle reduces to

$$\begin{array}{ccc} & A & 9 \\ & \times & 9 \\ \hline A & A & 1 \\ \hline \end{array}$$

Now, $9 \times 9 = 81$. We write 1 and
8 is carried forward Now $A \times 9 = A$
We choose $A = 4$

$\therefore$ $B = 9$, $A = 4$ is the required solution.

$$\begin{array}{ccc} & 4 & 9 \\ & \times & 9 \\ \hline 4 & 4 & 1 \\ \hline \end{array}$$

3. Find the value of M in

$$\begin{array}{cccc} & 3 & 1 & M \\ + & 1 & M & 3 \\ \hline & 4 & 6 & 8 \\ \hline \end{array}$$

If we add M and 3, we get a number 8 in the ones place. It means that $M = 5$. When we add M and 3, we get 8.

Next, if 1 and M are added, we get a number 6. It means that $M = 5$.
Hence, $M = 5$ is the correct solution.

Chapter-9 : Comparing Quantities

Exercise-1

1. Simplify the following ratios :

(a) 75 Paise : 3 rupees

$$1 \text{ rupee} = 100 \text{ paise}$$
$$= \frac{75 \text{ Paise}}{3 \text{ rupees}} = \frac{75}{3 \times 100}$$
$$= \frac{75}{300} = \frac{1}{4} = 1:4$$

(b) 2 hours 10 min : 26 min

$$1 \text{ hour} = 60 \text{ min}$$
$$\frac{2 \text{ hours } 10 \text{ min}}{26 \text{ min}} = \frac{2 \times 60 + 10}{26} = \frac{120 + 10}{26} = \frac{130}{26}$$
$$= \frac{5}{1} = 5:1$$

(c) 8 months : 3 years

$$1 \text{ year} = 12 \text{ months}$$
$$\frac{8 \text{ months}}{3 \text{ years}} = \frac{8}{3 \times 12} = \frac{8}{36} = \frac{2}{9}$$
$$= 2:9$$

(d) 450 m : 1 km 350 m

$$1 \text{ km} = 1000 \text{ m}$$
$$\frac{450 \text{ m}}{1 \text{ km } 350 \text{ m}} = \frac{450}{(1000 + 350)} = \frac{450}{1350} = \frac{1}{3} = 1:3$$

2. Express the following :

(a) $\frac{4}{25}$ as a percentage

$$\frac{4}{25} \times 100 = 4 \times 4 = 16\%$$

(b) 25% as a faction

$$25 \times \frac{1}{100} = \frac{25}{100} = \frac{1}{4}$$

(c) $\frac{42}{50}$ as a percentage

$$\frac{42}{50} \times 100 = 42 \times 2 = 84\%$$

(d) $2\frac{7}{8}\%$ as a decimal

$$\frac{23}{8}\% = \frac{23}{8 \times 100}$$

$$= \frac{23}{800} = 0 \cdot 02875$$

3. Divide ₹ 8100 in the ratio of 7 : 2.

$$7x + 2x = 8100$$

or $\quad 9x = 8100$

$\therefore \quad x = \frac{8100}{9} = 900$

Hence $\quad 7x = 7 \times 900 = ₹\,6300$

and $\quad 2x = 2 \times 900 = ₹\,1800$

First party is ₹ 6300 and second part is ₹ 1800.

4. Convert the following ratios to percentages :

(a) 5 : 12

$$\frac{5}{12} \times 100 = \frac{500}{12} = 41 \cdot 66 = 41 \cdot 7\%$$

(b) 7 : 8

$$\frac{7}{8} \times 100 = \frac{700}{8} = 87 \cdot 5\%$$

(c) 11 : 15

$$\frac{11}{15} \times 100 = \frac{1100}{15} = 73 \cdot 3\%$$

5. Mr. Mehra spends 30% of his monthly income on food, 10% on charity , saves 25% and is left with ₹ 15,750. What is his monthly salary? Also find his monthly savings.

Solution : Mr. Mehra spends on food = 30%

Mr. Mehra saves = 25%

Let Mr. Mehra's monthly salary be ₹ x.

$$30\% \text{ of } x + 10\% \text{ of } x + 25\% \text{ of } x + 15750 = x$$

$$\frac{30x}{100} + \frac{10x}{100} + \frac{25x}{100} + 15750 = x$$

$$0 \cdot 3x + 0 \cdot 1x + 0 \cdot 25x + 15750 = x$$

$$0 \cdot 35x = 15750$$

$$x = \frac{15750}{0 \cdot 35}$$

$$x = 45000$$

The monthly income of Mr. Mehra is ₹ 45000

Monthly savings = 25% of 45000

$$= \frac{25}{100} \times 45000$$

$$= ₹\,11250$$

6. In a class of 35 students, 60% are interested in Mathematics. How many students are not interested in Mathematics?

Solution : Total students of class = 35

Students interested in Mathematics = 60%

$$60\% \text{ of } 35 = \frac{60}{100} \times 35$$

$$= \frac{6 \times 35}{10} = \frac{210}{10} = 21$$

21 students are interested in Mathematics.

$\therefore$ $35 - 21 = 14$ students are not interested Mathematics.

7. The boys and girls in a school are in the ratio 8 : 5. If the number of boys is 168, what is the total strength of the school?

Solution : Let the no. of boys in the school be $8x$ and no. of girls be $5x$.

Total no. of boys = 168

So, $168 = 8x$, $x = \frac{168}{8}$

$$x = 21$$

The no. of girls = $5x = 5 \times 21 = 105$

No. of boys = $8x = 8 \times 21 = 168$

Total no. of students = no. of boys + no. of girls

$= 168 + 105$

$= 273$

8. Paurav wanted to buy a TV. He went to the shop with ₹ 12,000 and found that the model he liked was expensive. Also, the amount he had was only 80% of the cost of the TV. What was the cost of the TV?

Solution : As per question,

80% of the cost of TV = ₹ 12000

$\therefore$ 100% of the cost of TV = $\frac{100 \times 12000}{80}$ = ₹ 15000

9. An ice-cream vendor has vanilla and chocolate ice-creams in the ratio of 4 : 9. If there are 100 vanilla ice-creams, find the number of chocolate ice-creams.

Solution : Let the total no. of vanilla ice-creams be $4x$ and chocolate ice-creams be $9x$

Total no. of vanilla ice-creams = 100

So, $100 = 4x$, or $x = \frac{100}{4} = 25$

The no. of chocolate ice-creams = $9x = 9 \times 25$

$= 225$

10. ₹ 35000 is to be shared among three people so that the first person gets 50% of the second, who in turn gets 50% of the third. How much will each of them get?

Solution : Let the share of the third person be ₹ x

$\therefore$ Second person gets 50% of $x = \frac{50}{100} \times x = \frac{x}{2}$

$\therefore$ First person gets 50% of $\frac{x}{2} = \frac{50}{100} \times \frac{x}{2} = \frac{x}{4}$

$\therefore \quad x+\frac{x}{2}+\frac{x}{4}=35000$

$\therefore \quad 4x+2x+x=35000\times 4$

$\therefore \quad 7x=35000\times 4$

$\therefore \quad x=\frac{35000\times 4}{7}$

$=20000$

$\therefore$ Share of the first person $=\frac{20000}{4}=₹\,5000$

Share of the second person $=\frac{20000}{2}=₹\,10000$

Share of third person $= x = ₹\,20000$

Exercise-2

1. Neelam went to school for 192 days in a year. If her attendance was 75%, find the total number of working days of the school in that year.

Solution : Neelam's attendance was 192 days. Now, to find the total of working days of the school in that year, we have

So, the number of working days of the school in that year = 256 days.

2. Radha's earnings is 40% of Nimmi's. Nimmi's earnings is 25% of Seeta's earnings. If Seeta's earnings is ₹ 1,00,000, then find Radha's and Nimmi's earnings.

Solution : Seeta's earnings = ₹ 100,000

Nimmi's earning = 25% of Seeta's earnings

$= 25\%$ of 100,000

$=\frac{25}{100}\times 100{,}000$

$=25\times 1000=₹\,25000$

Radha's earnings = 40% of Nimmi's earnings

$= 40\%$ of 25000

$=\frac{40}{100}\times 25000$

$=10{,}000$

Hence, Nimmi's earnings = ₹ 25000 and Radha's earnings = ₹ 10,000

As per question,

Neelam's attemdemce was 75% of total workings days = 192

$\therefore$ 100% of total workings days Q=otal workings days $=\frac{100\times 192}{75}$

$=256$ days

3. A number is increased by 10% and then decreased by 10%. Find the net decrease percent.

Solution : Percehtage change formula when a number is first increased by x% and then decreased by y%.

$\{x-y-(xy/100)\}\%$

Here $x=10\%$ and $y=10\%$

Percentage change $= \{10 - 10 - (10 \times 10)/100\} = 1$
Thus, there is 1% decrease.

4. Alka has 15% more sweets than Priya. By what percent is Priya's number of sweets less than that of Alka's?

Solution : Let Priya'sweets be x. Then, 15% of Priya's sweets $= \frac{15}{100} \times x = \frac{3x}{20}$

Hence, Alka's sweets $= x + \frac{3x}{20} = \frac{20x + 3x}{20}$

$= \frac{23x}{20}$

Hence, the percent by which Priya's is less than Alka's $= \frac{\text{Sum of sweets}}{\text{Alka's sweets}} \times 100\%$

$$= \frac{\frac{3x}{20}}{\frac{23x}{20}} \times 100\% = \frac{3x}{23x} \times 100 = \frac{300}{23}$$

$= 31 \cdot 04\%$

5. The value of a machine decreases every year by 5%. If the present value of the machine is ₹ 27,500 what will be the value of the machine after 2 years?

Solution : Present value of the machine = ₹ 27500

Decrease percent per year = 5%
Actual decrease in the first year = 5% of ₹ 27500

$= 5\% \times 27500$

$= \frac{5}{100} \times 27500 =$ ₹ 1375

Machine value after one year = ₹ $(27500 - 1375)$

= ₹ 26,125

Actual decease in the second year = 5% of 26,125

$= 5\% \times 26{,}125$

$= \frac{5}{100} \times 26{,}125 = \frac{1}{20} \times 26125$

$= 1306 \cdot 25$

Machine value after second year.

= ₹ $(26{,}125 - 1306 \cdot 25)$

= ₹ $24818 \cdot 75$

6. The salary of A is 20% more than that of B. By what percent is B's salary less than that of A's salary?

Solution : Let B's salary = ₹ 100

$\therefore$ A's salary = ₹ (100 + 20% of 100) = ₹ 120

Now,
If, A's salary is ₹ 120, then B's salary = ₹ 100
If A's salary is ₹ 1, then B's salary = ₹ $\frac{100}{120}$

If A's salary is ₹100, then B's salary = ₹ $\left(\frac{100}{120} \times 100\right) = ₹ 83\frac{1}{3}$

Hence, B's salary is $\left(100 - 83\frac{1}{3}\right)$%. less than A's salary. B's salary is $16\frac{2}{3}$%. Less than A's salary.

7. Vijay receives a conveyance allowance of ₹ 800 per week from his company. If was reduced by 15%, find his new allowance.

Solution : Vijay's conveyance allowance = ₹ 800

His new allowance = 800 − 15% of 800

$$= 800 - 800 \times \frac{15}{100}$$

$$= 800\left(1 - \frac{15}{100}\right)$$

$$= 800\left(\frac{100-15}{100}\right)$$

$$= 800 \times \frac{85}{100}$$

$$= ₹ 680$$

8. Hemant's salary has been increased by 25%. By how much percent must his new salary now be reduced so that he gets his original salary?

Solution : Let his original salary = 100

New salary = 25% of 100 = 125

Salary to be reduced = $\frac{25 \times 100}{125} = 20\%$

9. Armaan's monthly salary is ₹ 25,500. If every year his salary is increased by 5%, what will be his monthly salary after two years?

Soluiton : Armaan's present salary = ₹ 25,500

Increase percent per year = 5%

Actual increase in the first year = 5% of ₹ 25,500

$$= ₹\left(\frac{5}{100} \times 25{,}500\right) = ₹ 1275$$

Armaan's salay after one year = ₹ (25,500 + 1275) = ₹ 26,775

Increase in the salary in the second year = 5% of ₹ 26,775

$$= ₹\left(\frac{5}{100} \times 26{,}775\right) = ₹ 1338 \cdot 75$$

Armaan's sawlary after two years = ₹ (26,775 + 1338 · 75) = ₹ 28,113 · 75

10. An incentive amount of ₹ 58,000 was divided among three salespersons, Lovleen, Ankita and Arun. Lovleen got 80% of what Ankita got and Ankita got 25% pf what Arun got. Find the amount each salesperson got. Also find what percentage of the incentive amount did Ankita receive.

Solution : Let Arun's incentive amount = ₹ x

∴ Ankita's incentive amount = 25% of x

$$= x \times \frac{25}{100} = \frac{1}{4}x$$

Lovleen's incantive amount = 80% of $\left(\frac{1}{4}x\right)$

$$= \frac{1}{4}x \times \frac{80}{100} = \frac{1}{5}x$$

As per question,

$$x + \frac{1}{4}x + \frac{1}{5}x = 58000$$

or $$\frac{20x + 5x + 4x}{20} = 58000$$

or $$\frac{29x}{20} = 58000$$

or $$x = \frac{58000 \times 20}{29} = 40000$$

Arun's incentive amount = ₹ 40000

Ankita's incentive amount = $40000 \times \frac{1}{4}$ = ₹10000

Lovleen's incentive amount = $40000 \times \frac{1}{5}$ = ₹ 8000

11. A man loses 20% of his money. After spending 25% of the remainder, he has ₹ 480 left. How much money did he originally have?

Solution : We will use x to represent the unknown. To find the remainder after 25% is spent, we know that 75% of the remainder is ₹ 480.

$$480 = 0 \cdot 75x$$

or $$x = \frac{480}{0 \cdot 75} = 640$$

So ₹ 640 is the remainder from the original amount. Now we also know that after spending 20% of the original amount, 80% of that amount is ₹ 640.

$$640 = 0 \cdot 80x$$

$$x = \frac{640}{0 \cdot 80}$$

$$x = 800$$

∴ ₹ 800 was the original amount.

12. The population of a town in the year 2012 was 1,32,300. If the population of the town increases every year 5%,what was the population of the town in the year 2010?

Solution : Let population is 2010 = P

Population in 2012 = 132300

As per question, by formula,

$$132300 = P\left(1 + \frac{5}{100}\right)^2$$

or $$132300 = P\left(1 + \frac{1}{20}\right)^2 = P\left(\frac{21}{20}\right)^2$$

or $$P = 132300 \times \frac{20}{21} \times \frac{20}{21}$$

$$= 120000$$

Chapter-10 : Commercial Mathematics

Exercise-1

1. If a shopkeeper sells an article for ₹ 2640, he incurs a loss of 12%. What should the selling price be in order to gain 12%?

Solution : SP of an article = ₹ 2640

$$\text{Loss }\% = 12\%$$

$$CP \text{ of the article} = \frac{100}{100 - L\%} \times SP$$

$$= \frac{100}{100-12} \times 2640$$

$$= \frac{100}{88} \times 2640$$

$$= 100 \times 30 = ₹\,3000$$

Now, CP = ₹ 3000, gain = 12%

$$\text{Expected } SP = \frac{(100 + P\%)}{100} \times CP$$

$$= \frac{100+12}{100} \times 3000$$

$$= \frac{112}{100} \times 3000 = ₹\,3360$$

Hence, to get a gain of 12%, the shopkeeper must sell the article for ₹ 3360.

2. By selling a car for ₹ 1,10,400, Sagar loses 8%. For how much should he sell his car so as to gain 8% ?

Solution : SP of a car = ₹ 1,10,400

$$CP \text{ of car} = \frac{100}{100 - L\%} \times SP$$

$$= \frac{100}{100-8} \times 110400$$

$$= \frac{100}{92} \times 110400$$

$$= ₹\,1{,}20{,}000$$

Now, CP = ₹ 1,20,000 and gain% = 8%

$$\text{Expected } SP = \frac{(100 + P\%)}{100} \times CP$$

$$= \frac{(100+8)}{100} \times 120000$$

$$= \frac{108}{100} \times 120000$$

$$= ₹\,1{,}29{,}600$$

Hence, to get a gain of 8% Sagar must sell the car for ₹ 1,29,600

3. Rajesh bought a scooter for ₹ 21,000. He sold it to Sahil at a loss of 5%. Sahil spent ₹ 2450 on the repairs and sold the scooter to Chahat at a profit of 8%. How much did Chahat pay for the scooter?

Solution : CP of a scooter bought by Rajesh

$$CP = ₹\,21{,}000$$

$$\text{Loss }\% = 5\%$$

$$SP \text{ of scooter} = \frac{(100 - L\%)}{100} \times CP$$

$$= \frac{(100-5)}{100} \times 21000$$

$$= \frac{95}{100} \times 21000$$

$$= ₹\,19{,}950$$

∴ Rajesh sold the scooter to Sahil for ₹ 19,950. Money spent on repairing = ₹ 2450

So, actual CP = ₹ (19950 + 2450)

$$= ₹\,22{,}400$$

$$\text{Profit }\% = 8\%$$

$$\text{Expected } SP = \frac{(100 + P\%)}{100} \times CP$$

$$= \frac{(100+8)}{100} \times 22400$$

$$= \frac{108}{100} \times 22400$$

$$= ₹\,24{,}192$$

Hence, Chahat paid for the scooter for ₹ 24,192.

4. 4% more is gained by selling a foot mat for ₹ 180 than by selling it for ₹ 175. Find the cost price of the mat.

Actual SP of a foot mat = ₹ 175

Let actual Profit % = x%

$$\therefore \quad CP = \frac{100}{100 + P\%} \times SP$$

$$= \frac{100}{100 + x} \times 175$$

And if SP of a foot mat = ₹ 180

then profit % = $(x + 4)$%

$$\therefore \quad CP = \frac{100}{100 + P\%} \times SP$$

$$= \frac{100}{100 + x + 4} \times 180$$

$$= \frac{100 \times 180}{104 + x}$$

Now, $$\frac{100}{100 + x} \times 175 = \frac{100}{104 + x} \times 180$$

$$(104+x)175 = (100+x)180$$
$$(104+x)35 = (100+x)36$$
$$3640+35x = 3600+36x$$
$$x = 40$$

So, actual profit % = 40%

actual SP = ₹175

$$\therefore \quad CP = \frac{100}{100+P\%} \times SP$$
$$= \frac{100}{100+40} \times 175$$
$$= ₹125$$

Hence, the cost price of the mat is ₹125.

5. A shopkeeper sells two suitcases for ₹2464 each, gaining 12% on one and losing 12% on the other. Find his gain or loss percent on the whole.

Solution : SP of first suitcases = ₹2464

gain % = 12%

$$\therefore \quad CP = \frac{100}{100+P\%} \times SP$$
$$= \frac{100}{100+12} \times 2464$$
$$= \frac{100}{112} \times 2464 = ₹2200$$

SP of other suitcase = ₹2464

Loss % = 12%

$$\therefore \quad CP = \frac{100}{100-L\%} \times SP$$
$$= \frac{100}{100-12} \times 2464$$
$$= \frac{100}{88} \times 2464 = ₹2800$$

So, Total SP = ₹ (2464 + 2464) = ₹ 4,928

Total CP = ₹ (2200 + 2800) = ₹ 5,000

Here $SP < CP$

So, Loss = $CP - SP$

= ₹ (5,000 − 4,928) = ₹ 72

$$\text{Overall Loss }\% = \frac{\text{Loss}}{\text{CP}} \times 100$$
$$= \frac{72}{5000} \times 100\% = \frac{72}{50} = \frac{36}{25}$$
$$= 1\frac{11}{25}\%$$

6. Rohit purchased a shirt for ₹ 2700 inclusive of 8% VAT. Find the price of the shirt before the VAT.
Let CP

Solution : Let CP of a shirt = ₹ x

CP of a shirt included VAT = ₹ 2700

VAT % = 8%

Amount paid = $x + 8\%$ of x

$$= \left(1 + \frac{8}{100}\right)x = \frac{108}{100}x$$

$$\therefore \qquad \frac{108x}{100} = 2700$$

$$x = \frac{2700 \times 100}{108}$$

$$x = 2500$$

Hence, the cost price of the shirt before the VAT is ₹ 2500.

7. Jayesh bought a handbag for his mother for ₹ 5400 including 8% VAT. Find the price before the VAT was levied on it.

Solution : Let actual CP of a handbag = ₹ x

CP of a handbag included VAT = ₹ 5400

VAT % = 8%

Amount paid = $x + 8\%$ of x

$$= \left(1 + \frac{8}{100}\right) x = \frac{108}{100}$$

$$\therefore \qquad \frac{108x}{100} = 5400$$

$$x = \frac{5400 \times 100}{108} = 5000$$

Hence, the price before the VAT is ₹ 5000.

8. A toaster is marked for sale at ₹ 1650. There is a discount of 8% and still the profit made is 20% of the cost price of the toaster. Find its cost price.

Solution : MP of a toaster = ₹ 1650

discount % = 8%

$\therefore$ discount = 8% of ₹ 1650

$$= 8 \times \frac{1}{100} \times 1650$$

= ₹ 132

$\therefore$ Amount = ₹ (1650 − 132)

= ₹ 1518

Now, given that

Amount = $CP + 20\%$ of CP

$$1518 = CP + \frac{20}{100} CP$$

$$1518 = \frac{6}{5} CP$$

$$CP = \frac{1518 \times 5}{6} = 1265$$

Hence, the cost price of toaster is ₹ 1265.

9. Kajal gives a discount of 15% on the sarees sold by her and still gets a profit of $13\frac{1}{3}$ %. What is the cost price of a saree whose marked price is ₹ 3200?

Solution : MP of a saree = ₹ 3200

discount % = 15%

$$\text{discount} = 3200 \times \frac{15}{100} = ₹\ 480$$

$\therefore$ SP = ₹ (3200 − 480) = ₹ 2720

$$\text{Profit } \% = 13\frac{1}{3}\% = \frac{40}{3}\%$$

$$\therefore \quad CP = \frac{100}{100 + P\%} \times SP$$

$$= \frac{100}{\left(100 + \frac{40}{3}\right)} \times 2720$$

$$= \frac{100 \times 3}{340} \times 2720 = ₹\ 2400$$

Hence, the cost price of saree is ₹ 2400.

10. A shopkeeper marks his goods at such a price that after allowing $12 \cdot 5\%$ discount for each payment, he still gains 5%. Find the marked price of an article which costs him ₹ 500.

Solution : Let $M.P.$ of an article = ₹ x

distcount = $12 \cdot 5\%$ of x

$$= \frac{x}{8}$$

$$\text{then amount} = ₹\left(x - \frac{x}{8}\right) = ₹\ \frac{7x}{8}$$

Now, Amount = CP + 5% of CP

$$\frac{7}{8}x = 500 + 5 \times \frac{1}{100} \times 500$$

$$\frac{7}{8}x = 500 + 25$$

$$\frac{7}{8}x = 525$$

$$x = \frac{525 \times 8}{7} = 600$$

Hence, The marked price of the article is ₹ 600.

11. Sachin bought an article for ₹ 240 and sold it at a bill amount of ₹ 316 · 80 which includes 10% *VAT*. Find his profit or loss percent.

Solution : Let *MP* of an artile = ₹ x

CP of the article = ₹ 240

SP of thr article included VAT = 316 · 80

VAT % = 10% of x

So, $316 \cdot 80 = x + 10\% \text{ of } x$

or $316 \cdot 80 = x + \frac{10x}{100}$

$$\frac{31680}{100} = \frac{110x}{100}$$

$$x = \frac{3168}{11} = ₹\,288$$

∴ $SP = ₹\,288$

here $SP > CP$

$$\text{Profit} = SP - CP$$

$$= ₹\,(288 - 240) = ₹\,48$$

$$\text{Profit \%} = \frac{P}{CP} \times 100$$

$$= \frac{48}{240} \times 100 = 20\%$$

12. An article is available for ₹ 14300 inclusive of *VAT* at the rate of 10%. Find its marked price. What will be its price of the rate of *VAT* changes to 12%?

Solution : Let MP of article = ₹ x

SP of an article included VAT = ₹ 14300

VAT % = 10%

∴ $14300 = x + 10\% \text{ of } x$

$$14300 = x + \frac{10}{100} \times x$$

$$14300 = x + \frac{1}{10}x = \frac{11}{10}x$$

$$x = \frac{14300 \times 10}{11} = ₹\,13000$$

So, the marked price of an article is ₹ 13000.

Now, if VAT% = 12%

$M.P. = ₹\,13000$

∴ Price will be = 13000 + 12% of 13000

$$= 13000 + 12 \times \frac{13000}{100}$$

$$= 13000 + 1560$$

$$= ₹\,14{,}560$$

13. The cost price of 10 pens is the selling price of 8 pens. Find the gain or loss percent.

Solution : Given

Cost price of 10 pens = Selling price of 8 pens

$$\text{Profit } \% = \frac{\text{Profit}}{CP} \times 100 \qquad [\text{Profit} = SP - CP]$$

$$= \frac{10-8}{8} \times 100$$

$$= \frac{2}{8} \times 100 = 25\%$$

14. A trader sold two fans at ₹1980 each. If he gets a profit of 10% on one and a loss of 10% on the other, find his overall gain or loss percent.

Solution : SP of first fan = ₹1980

Profit % = 10%

$$\therefore \quad CP = \frac{100}{100 + P\%} \times SP$$

$$= \frac{100}{100+10} \times 1980$$

$$= \frac{100}{110} \times 1980$$

$$= ₹1800$$

SP of second fan = ₹1980

Loss% = 10%

$$\therefore \quad CP = \frac{100}{100 - L\%} \times SP$$

$$= \frac{100}{100-10} \times 1980$$

$$= \frac{100}{90} \times 1980$$

$$= ₹2200$$

So, Total CP = ₹ (1800 + 2200) = ₹ 4,000

Total SP = ₹ (1980 + 1980) = ₹ 3,960

Here, $CP > SP$

Loss = $CP - SP$

= ₹ (4000 − 3960) = ₹ 40

$$\text{Loss } \% = \frac{\text{loss}}{CP} \times 100$$

$$= \frac{40}{4000} \times 100 = 1\%$$

Overall loss % = 1%

Exercise-2

1. Find the compound interest on ₹ 5,000 for 3 years at 5%. Also find the simple interest for ₹ 5,000 for 3 years at 6%. Find the difference in interests.

Solution : $P = ₹\ 5{,}000$, $R = 5\%$, $n = 3$ years

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 5000 \times \left(1+\frac{5}{100}\right)^3$$

$$= 5000 \times \left(1+\frac{1}{20}\right)^3$$

$$= 5000 \times \left(\frac{21}{20}\right)^3$$

$$= 5000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$A = ₹\ 5788 \cdot 125$$

Compound interest = Amount – Principal

$$= ₹\ (5788 \cdot 125 - 5000)$$

$$= ₹\ 788 \cdot 125$$

Now, $P = ₹\ 5000$, $T = 3$ years, $R = 6\%$

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{5000 \times 3 \times 6}{100} = ₹\ 900$$

$\therefore$ Difference in interests $= ₹\ (900 - 788 \cdot 125)$

$$= ₹ 111 \cdot 875$$

$$\cong ₹ 111 \cdot 88$$

2. Find the compound interest on ₹10,000 for 1 year and 3 months at 8% per annum compounded annually.

Solution : $P = ₹ 10{,}000$ $R = 8\%$, $n = 1$ year 3 months

First, we will be using the formula for finding the amount after one year and then finding the interest for 3 months, separately.

$\therefore$ $$A = P\left(1+\frac{R}{100}\right)^n$$

$$= ₹ 10{,}000\left(1+\frac{8}{100}\right)^1$$

$$= ₹ 10{,}000 \times \frac{108}{100} = ₹ 10{,}800$$

Now, for 3 months, $T = \frac{3}{12}$ year $= \frac{1}{4}$ year

$$P = ₹ 10{,}800,\ R = 8\%$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{10800 \times 8 \times 1}{100 \times 4}$$

$$= ₹\,216$$

Amount after 1 year 3 months $= ₹\,(10800 + 216)$

$$= ₹\,11{,}016$$

$\therefore$ Compound interest $= A - P$

$$= ₹ = (11{,}016 - 10{,}000)$$

$$= ₹\,1{,}016$$

3. Find the amount and compounded interest if a sum of ₹ 4000 is borrowed for $2\frac{1}{2}$ years at the rate of 8% per annum.

Solution : $P = ₹\,4000, \quad R = 8\%, \quad n = 2\frac{1}{2}$ years

First, we will use the formula for finding the amount after two years and then find the interest for half a year separately.

$$\text{Amount } (A) = P\left(1 + \frac{R}{100}\right)^n$$

$$= ₹\,4000\left(1 + \frac{8}{100}\right)^2$$

$$= ₹\,4000 \times \frac{108}{100} \times \frac{108}{100}$$

$$= ₹\,4665 \cdot 6$$

Now, for the half year, we have

$$P = ₹\,4665 \cdot 6, \qquad R = 8\%, \quad T = \frac{1}{2} \text{ year}$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{4665 \cdot 6 \times 8 \times 1}{100 \times 2} = ₹\,186 \cdot 62$$

$\therefore$ Amount after $2\frac{1}{2}$ years $= ₹\,(4665 \cdot 6 + 186 \cdot 62)$

$$= ₹\,4852 \cdot 2$$

$$C.I. = A - P$$

$$= ₹\,(4852 \cdot 2 - 4000) = ₹\,852 \cdot 2$$

4. Babita borrowed ₹ 15,000 from a bank to Purchase a TV set. If the bank charges interest quarterly at the rate of 8% per annum, what is the amount she has to pay at the end of 9 months to clear her debt?

Solution :

$$P = ₹\,15000, \quad R = 8\%, \quad n = \frac{9}{12} \text{ year}$$

$\therefore \quad A = P\left(1+\frac{R}{4\times 100}\right)^{4n}$ (Quarterly)

$$= ₹15000\left(1+\frac{8}{4\times 100}\right)^{4\times\frac{9}{12}}$$

$$= ₹15000\left(1+\frac{2}{100}\right)^{3}$$

$$= ₹15000\left(\frac{102}{100}\right)^{3}$$

$$= ₹15000\times\frac{102}{100}\times\frac{102}{100}\times\frac{102}{100}$$

$$= ₹15918\cdot 12$$

5. Ruhi invested ₹ 25,000 at the rate of 12% per annum compounded half yearly. What amount would he get after a year ?

Solution : $P = ₹\,25000,\quad R = 12\%,\ n = 1$ year

$\therefore \quad \text{Amount} = P\left(1+\frac{R}{2\times 100}\right)^{2n}$ (half yearly)

$$= ₹\,25000\left(1+\frac{12}{2\times 100}\right)^{2\times 1}$$

$$= ₹\,25000\left(\frac{106}{100}\right)^{2\times 1}$$

$$= ₹\,25000\times\left(\frac{106}{100}\right)^{2}$$

$$= ₹\,25000\times\frac{106}{100}\times\frac{106}{100}$$

$$= ₹\,28090$$

6. Find the sum if the amount at the end of 2 years at 10% compounded annually is ₹ 7986.

Solution : $n = 2$ years, $R = 10\%,\ A = ₹\ 7986$ $P = ?$

$$\text{Amount } (A) = P\left(1+\frac{R}{100}\right)^{n}$$

$$7986 = P\left(1+\frac{10}{100}\right)^{2}$$

$$7986 = P\left(\frac{11}{10}\right)^{2}$$

$$P = \frac{7986\times 10\times 10}{11\times 11}$$

$$= ₹\,6600$$

7. Find the rate of compound interest per annum at which ₹ 12500 will amount to ₹ 15680 in two years.

Solution : $P = 12500$, $A = ₹15680$, $n = 2$ years

$$R = ?$$

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$15680 = 12500\left(1+\frac{R}{100}\right)^{2}$$

$$\frac{15680}{12500} = \left(1+\frac{R}{100}\right)^{2}$$

$$\frac{1568}{1250} = \left(1+\frac{R}{100}\right)^{2}$$

$$\frac{784}{625} = \left(1+\frac{R}{100}\right)^{2}$$

$$\left(\frac{28}{25}\right)^{2} = \left(1+\frac{R}{100}\right)^{2}$$

$\therefore$ $$1+\frac{R}{100} = \frac{28}{25}$$

$$\frac{R}{100} = \frac{28}{25}\,\frac{1}{-1}$$

$$\frac{R}{100} = \frac{3}{25}$$

or $$R = \frac{3}{25}\times 100$$

$$R = 12\%$$

8. Find the amount and CI for ₹100 at the rate of 10% compounded half yearly for $1\frac{1}{2}$ years.

Solution : $P = ₹100$, $R = 10\%$, $n = 1\frac{1}{2}$ years $= \frac{3}{2}$ years

$$A = P\left(1+\frac{R}{2\times 100}\right)^{2n}$$

$$= ₹100\left(1+\frac{10}{2\times 100}\right)^{2\times\frac{3}{2}}$$

$$= ₹100\left(1+\frac{5}{100}\right)^{3}$$

$$= ₹100\times\frac{105}{100}\times\frac{105}{100}\times\frac{105}{100}$$

$$= ₹115\cdot 7625$$

$$C.I. = A - P$$

$$= ₹\,(115\cdot 7625 - 100)$$

$$= ₹15\cdot 7625$$

9. Janu took a loan of ₹ 60,000 from a bank. If the rate of interest is 8% per annum, find the difference in the amount she will be paying after 1 year if the interest is

(a) Compounded half yearly

(b) Compounded quarterly

Solution : $P = ₹\ 60{,}000, \quad R = 8\%, \quad n = 1\,\text{year}$

(a) For compounded half yearly

$$A = P\left(1 + \frac{R}{2 \times 100}\right)^{2n}$$

$$= ₹\ 60{,}000\left(1 + \frac{8}{2 \times 100}\right)^{2 \times 1}$$

$$= ₹\ 60{,}000\left(1 + \frac{4}{100}\right)^{2}$$

$$= ₹\ 60{,}000 \times \frac{104}{100} \times \frac{104}{100}$$

$$A = ₹\ 64{,}896$$

(b) For compounded quarterly

$$A = P\left(1 + \frac{R}{4 \times 100}\right)^{4n}$$

$$= ₹\ 60{,}000\left(1 + \frac{8}{4 \times 100}\right)^{4 \times 1}$$

$$= ₹\ 60{,}000\left(1 + \frac{2}{100}\right)^{4}$$

$$= ₹\ 60{,}000 \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100}$$

$$A = ₹\ 64945 \cdot 9296$$

$$\cong ₹\ 64945 \cdot 93$$

10. Find the time taken for a sum of ₹ 8000 to amount ₹ 9261 if the rate of interest is 5% per annum compounded annually.

Solution : $n = ?, \quad P = ₹\ 8000, \quad A = ₹\ 9261, \quad R = 5\%$

$$A = P\left(1 + \frac{R}{100}\right)^{n}$$

$$9261 = 8000\left(1 + \frac{5}{100}\right)^{n}$$

$$\frac{9261}{8000} = \left(\frac{105}{100}\right)^{n}$$

$$\left(\frac{21}{20}\right)^{3} = \left(\frac{21}{20}\right)^{n}$$

$$n = 3 \text{ years}$$

11. The value of a residential flat constructed at a cost of ₹ 10,00,000 is depreciating at the rate of 10% per annum. What will be its value 3 years after construction?

Solution : $P = ₹\,10{,}00{,}000$, Reduction $(R) = 10\%$, $n = 3$ years

$$A = P\left(1 - \frac{R}{100}\right)^n$$

$$= ₹\,10{,}00{,}000\left(1 - \frac{10}{100}\right)^3$$

$$= ₹\,10{,}00{,}000 \times \left(\frac{9}{10}\right)^3$$

$$= ₹\,10{,}00{,}000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$\therefore$ $A = ₹\,7{,}29{,}000$

12. If the population of a town increased from 12500 to 13375 in one year, what is the rate of growth of the population?

Solution : $P = 12500$, $A = 13375$, $n = 1$ year $R = ?$

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$13375 = 12500\left(1 + \frac{R}{100}\right)^1$$

or $$\frac{13375}{12500} = \left(1 + \frac{R}{100}\right)$$

$$\frac{107}{100} = 1 + \frac{R}{100}$$

$$\frac{R}{100} = \frac{107}{100} - 1$$

$$\frac{R}{100} = \frac{7}{100}$$

$\therefore$ $R = 7\%$

13. Suraj lent ₹ 2000 at compound interest at 10% payable yearly, while Ankit lent ₹ 2000 at compound interest at 10% payable half-yearly. Find the difference in the interest received by Suraj and Ankit at the end of one year.

Solution : For Suraj : $P = ₹\,2000$, $R = 10\%$, $n = 1$ year

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= ₹\,2000\left(1 + \frac{10}{100}\right)^1$$

$$= ₹\,200 \times \frac{11}{10} = ₹\,2200$$

$$C.I. = A - P$$

$$= ₹\,(2200 - 2000) = ₹\,200$$

For Ankit

$$P = ₹\,2000, \quad R = 10\%$$

$$n = 1 \text{ year}$$

$$A = P\left(1 + \frac{R}{2 \times 100}\right)^{2n} \quad \text{(half-yearly)}$$

$$= ₹\,2000\left(1 + \frac{10}{2 \times 100}\right)^{2 \times 1}$$

$$= ₹\,2000 \times \left(1 + \frac{5}{100}\right)^{2}$$

$$= ₹\,2000 \times \left(\frac{105}{100}\right)^{2}$$

$$= ₹\,2000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= ₹\,2{,}205$$

$$C.I. = A - P$$

$$= ₹\,(2205 - 2000) = ₹\,205$$

Difference between their compound interest = ₹ (205 − 200) = ₹ 5

So, Ankit earred ₹ 5 more interest.

14. The compound interest on a certain sum of money amounts to ₹ 820 for 2 years at the rate of 5% per annum compounded annually. Find the sum.

Solution : $C.I. = ₹\,820$,

$n = 2$ years, $R = 5\%$, $P = ?$

$$CI = P\left[\left(1 + \frac{R}{100}\right)^{n} - 1\right]$$

$$820 = P\left[\left(1 + \frac{5}{100}\right)^{2} - 1\right]$$

$$820 = P\left[\left(\frac{105}{100}\right)^{2} - 1\right]$$

$$820 = P\left[\left(\frac{21}{20}\right)^{2} - 1\right]$$

$$820 = P\left[\frac{441}{400} - 1\right]$$

$$820 = P \times \frac{41}{400}$$

or $$P = \frac{820 \times 400}{41}$$

$$P = ₹\,8000$$

Chapter- 11 : Direct and Inverse Variation

Exercise-1

1. If 5 pens cost ₹ 70, how much will 12 pens cost?

Solution : Let required no. be y. Since no. of pens increases so the cost will also increase. It is the case of direct variation.

No. of pens (x)	5	12
Cost (in ₹) of pens (y)	70	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{5}{70} = \frac{12}{y}$$

or $$y = \frac{70 \times 12}{5} = 168$$

Hence, the cost of 70 pens is ₹ 168.

2. The bus fare for 112 km is ₹ 728. How much will be the fare for 240 km?

Solution : Let the required fare be y. Since distance increases so the fare will also increase So, the variation is direct.

Distance in km (x)	112	240
Fare in ₹ (y)	728	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{112}{728} = \frac{240}{y}$$

or $$y = \frac{728 \times 240}{112}$$

$$= 1{,}560$$

Hence, the fare for 240 km is ₹1560.

3. A machine takes 12 hours for cutting 240 tools. How many tools will be cut in 25 hours?

Solution : Let the required no. be y. Since the more hours, the more tools will be required. So it is the case of direct variation.

No. of hours (x)	12	25
No. of tools (y)	240	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{12}{240} = \frac{25}{y}$$

or $$y = \frac{240 \times 25}{12} = 500$$

Hence, 500 tools will be cut in 25 hours.

4. A tourist taxi charges ₹ 375 for travelling a distance of 150 km. Find the distance that can be travelled for ₹ $562 \cdot 5$ by the taxi.

Solution : Let the required no. be *y*. Since charges increase, so the distance will also increase. The variation is direct.

Taxi charges in ₹ (x)	375	$562 \cdot 5$
Distance in km (y)	150	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{375}{150} = \frac{562 \cdot 5}{y}$$

or $$y = \frac{150 \times 562 \cdot 5}{375} = 225 \text{ km}$$

Hence, the required distance is 225 km.

5. A worker is paid ₹ 450 for 6 days of work. If his total income for the same type of work is ₹ 1800, for how many days did he work?

Solution : Let the required no. be *y*. Since more income, more days will required. So it is the case of direct variation.

Income in ₹ (x)	450	1800
No. of days (y)	6	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{450}{6} = \frac{1800}{y}$$

or $$y = \frac{1800 \times 6}{450}$$

$\therefore$ $$y = 24$$

Hence, he did the work for 24 days.

6. If the thickness of 500 sheets of paper is $3 \cdot 5$ cm, what would be the thickeness of 275 sheets of paper?

Solution : Let the required thickness be *y*. Since less sheets for less thickness, so variation is direct.

No. of sheets (x)	500	275
Thickness in cm (y)	$3 \cdot 5$	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{500}{3\cdot5} = \frac{275}{y}$$

$$500 \times y = 275 \times 3\cdot5$$

$$\therefore \quad y = \frac{275 \times 3\cdot5}{500} = 1\cdot925 \text{ cm}$$

Hence, the thickness of 275 sheets is $1\cdot925$ cm.

7. Shashi types 540 words during half an hour. How many words would she type in 6 minutes?

Solution : Let the required no. be *x*. Since less time, so no. of words will also be less. In this case variation is direct.

No. of words (x)	540	x
Time in min (y)	30	6

So, $$x_1 y_2 = x_2 y_1$$

$$540 \times 6 = x \times 30$$

or $$x = \frac{540 \times 6}{30}$$

$$\therefore \quad x = 108$$

Hence, the required words are 108.

8. A car travels 330 km in 5 hours with a uniform speed. In how many hours will it travel 4290 km?

Solution : Let the required time be *y*. Since more distance, so it will take more time. The variation is direct.

Distance in km (x)	330	4290
Time in hours (y)	5	y

So, $$\frac{x_1}{y_1} = \frac{x_1}{y_2}$$

$$\frac{330}{5} = \frac{4290}{y}$$

$$\therefore \quad y = \frac{5 \times 4290}{330} = 65$$

Hence, the car will travels in 65 hours.

9. Saurabh can walk a distance of 210 m in 90 minutes. Find the time taken by him to cover a distance of 560 m.

Solution : Let the required time be *y*. Since more distance, more time will be taken. The variation is direct.

Distance in m (x)	210	560
Time in minutes (y)	90	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{210}{90} = \frac{560}{y}$$

$\therefore$ $$y = \frac{90 \times 560}{210} = 240$$

Hence, The required time is 240 min or 4 hours.

10. If a boat travels 125 km in $2\frac{1}{2}$ hours, how far will it travel in 12 hours?

Solution : Let the required time be *y*. Since time increases, distance will also increase. The variation is direct.

Time taken in hours (x)	$2\frac{1}{2} = \frac{5}{2}$	12
Distance travelled in km (y)	125	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{5}{2} \times \frac{1}{125} = \frac{12}{y}$$

$\therefore$ $$y = \frac{12 \times 2 \times 125}{5} = 600$$

Hence, the boat will travel 600 km in 12 hours.

11. The amount of extension is an elastic spring has a direct variation to the weight that is hanged from it. If a weight of 250 g produces an extension of $3 \cdot 25$ cm, then how much weight will produce an extension of $14 \cdot 3$ cm?

Solution : Let the required extension be *y*. Here the more is weight, the more will be extension. This is the case of direct variation.

Extension in cm (x)	$3 \cdot 25$	$14 \cdot 3$
Weight in gm (y)	250	y

So, $$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{3 \cdot 25}{250} = \frac{14 \cdot 3}{y}$$

$\therefore$ $$y = \frac{250 \times 14 \cdot 3}{3 \cdot 25} = 1100$$

Hence, the required weight is 1100 gm.

12. If 1350 perfume bottles of the same size can be packed in 50 cartons of the same size, how many such cartons are required to pack 2997 perfume bottes?

Solution : Let the required cartons be *y*. Since more bottles will require more cartons, so the variation is direct.

No. of bottles (x)	1350	2997
No. of cartons (y)	50	y

So, $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2}$

$$\frac{1350}{50} = \frac{2997}{y}$$

$\therefore$ $$y = \frac{2997 \times 50}{1350} = 111$$

Hence, 111 cartons are required to pack 2997 perfume bottles.

13. 26 labourers can build a $25 \cdot 4$ m long wall up to a certain fixed height in 1 day. How many labourers should be employed to build a wall of the same height but of length $63 \cdot 5$ m in 1 day?

Solution : Let the required labourers be y. Since the more length, the more labourers will be required, this is the direct variation.

Length in m (x)	$25 \cdot 4$	$63 \cdot 5$
No. of labourers (y)	26	y

So, $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2}$

$$\frac{25 \cdot 4}{26} = \frac{63 \cdot 5}{y}$$

$\therefore$ $$y = \frac{26 \times 63 \cdot 5}{25 \cdot 4} = 65$$

Hence, the required labourers are 65.

14. A pole of length 12 m casts a shadow of length $15 \cdot 6$ m.

(a) Find the length of the shadow cast by another pole of length 18 m.

(b) Find the height of a pole which casts a shadow of length $11 \cdot 7$ m.

Solution : (a) Let the required length be y. Since length of pole will increase with length of shadow, this is the direct variation.

Length of pole in m (x)	12	18
Length of shadow in m (y)	$15 \cdot 6$	y

So, $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2}$

$$\frac{12}{15 \cdot 6} = \frac{18}{y}$$

$\therefore$ $$y = \frac{15 \cdot 6 \times 18}{12} = 23 \cdot 4$$

Hence, the length of shadow is $23 \cdot 4$ m

(b) Let the required length be y. Since length of shadow will decease with length of pole, this is the direct vaiation.

Length of shadow in m (x)	$15 \cdot 6$	$11 \cdot 7$
Length of pole in m (y)	12	y

So, $x_1 y_2 = x_2 y_1$

$$15 \cdot 6 \times y = 12 \times 11 \cdot 7$$

$$y = \frac{12 \times 11 \cdot 7}{15 \cdot 6} = 9$$

Hence, the length of pole is 9 m.

15. Find the values of the unknowns if x and y vary directly.

Solution : (a) Since the variation is diect, we have $\frac{2}{10} = \frac{1}{5} = k$

Now, $\frac{P}{125} = \frac{1}{5}$

or $P = \frac{125}{5} = 25$

$P = 25$

(b) Since the variation is direct, we have

$$\frac{5}{15} = \frac{1}{3} = k$$

Now, $\frac{8}{a} = \frac{1}{3}$

or $a = 8 \times 3$

$a = 24$

And, $\frac{b}{72} = \frac{1}{3}$

or $b = \frac{72}{3}$

$\therefore$ $b = 24$

Exercise-2

1. Check if x and y vary inversely.

Solution : (a) Here, $x_1 y_1 \neq x_2 y_2 \neq x_3 y_3 \neq x_4 y_4$

So, x and y do not vary inversely.

(b) Here, $x_1 y_1 = x_2 y_2 = x_3 y_3 = x_4 y_4 = 100$

So, x and y vary inversely.

2. If 39 men can dig a canal in 15 days, in how many days can 45 men dig a canal whose length is double this canal?

Solution : Let the required days be y. Since less days will be required with more men, this is the inverse variation.

No. of men (x)	39	45
No. of day (y)	15	y

So, $x_1 y_1 = x_2 y_2$

$$39 \times 15 = 45 \times y$$

$$\therefore \quad y = \frac{39 \times 15}{45} = 13$$

Since, length of canal is double $= 13 \times 2 = 26$

$\therefore$ The required no. of days is 26.

3. In a hostel of 80 girls, there are food provisions for 25 days. If 30 girls leave the hostel, how long will these provision last?

Solution : Let the required no. of days be y. Since the no. of girls decreases, the no.of days will increase. It is the case of inverse variation.

No. of girls (x)	80	$80 - 30 = 50$
No. of days (y)	25	y

So, $x_1 y_1 = x_2 y_2$

$$80 \times 25 = 50 \times y$$

$$\therefore \quad y = \frac{80 \times 25}{50} = 40$$

Hence, the required days are 40.

4. If 28 pumps can empty a reservoir in 18 hours, how long will 35 pumps take to do the same work?

Solution : Let the required time be y. Since no. of pumps increase, the time will decrease. This is the inverse variation.

No. of pumps (x)	28	35
Time in hours (y)	18	y

So, $x_1 y_1 = x_2 y_2$

$$28 \times 18 = 35 \times y$$

$$y = \frac{28 \times 18}{35} = 14 \cdot 4$$

Hence, the required time is $= 14 \cdot 4$ hours.

5. 35 women can sow a field in 18 days. If the sowing is to be done in 15 days, how many more women will be required?

Solution : Let the required no. of women be x. Since days will decrease with more women, this is the inverse variation.

No. of women (x)	35	x
No. of days (y)	18	15

So, $x_1 y_1 = x_2 y_2$

$$35 \times 18 = x \times 15$$

$$x = \frac{35 \times 18}{15} = 42$$

So, more women will be required $= 42 - 35$

$= 7$ women

6. 4 pipes can fill a tank in 70 minutes. How long will it take to fill the tank if 7 pipes are used?
Solution : Let the required time be y. Here, no. of pipes increases, so the time will decrease. This is the inverse variation.

No. of pipes (x)	4	7
Time in minutes (y)	70	y

So, $x_1 y_1 = x_2 y_2$

$$4 \times 70 = 7 \times y$$

$$\therefore \quad y = \frac{4 \times 70}{7} = 40$$

Hence, the time taken is 40 minutes.

7. 45 cows can graze a field in 22 days. How many cows will graze the same field in 15 days?
Solution : Let the required cows be y. Since no. of days decreases, no. of cows will increase. This is the inverse variation.

No. of days (x)	22	15
No. of cows (y)	45	y

So, $x_1 y_1 = x_2 y_2$

$$22 \times 45 = 15 \times y$$

$$y = \frac{22 \times 45}{15} = 66$$

Hence, 66 cows will graze the field.

8. A train is moving at a speed of 100 km/h. It takes 18 minutes to reach a station. What should be its speed if the train wants to reach the station in 15 minutes?
Solution : Let the required speed be y km/h. Since time decreases, the speed will increase. so, the variation is inverse.

Time in minutes (x)	18	15
Speed of train in km/h (y)	100	y

So, $x_1 y_1 = x_2 y_2$

$$18 \times 100 = 15 \times y$$

$$\therefore \quad y = \frac{18 \times 100}{15} = 120$$

Hence, the required speed of train is 120 km/h.

9. A group of 210 students go camping with provisions for 60 days. After 10 days, 60 students leave. How long will the remaining provisions last?
Solution : Let the required days be y. Since no. of students decreases, the days will increase. So, the variation is inverse.

No. of students (x)	210	150
No. of days (y)	$60 - 10 = 50$	y

So, $x_1 y_1 = x_2 y_2$

$$210 \times 50 = 150 \times y$$

$\therefore$ $$y = \frac{210 \times 50}{150} = 70$$

Hence, the remaining provision will last for 70 days.

10. There is enough food for 20 animals on a farm for 6 days. If 10 more animals are added to the farm, how long will the food last?

Solution : Let the required days be y. Since no. of animals increases, the days will decrease. So the variation is inverse.

No. of animals (x)	20	$20+10=30$
No. of days (y)	6	y

So, $x_1 y_1 = x_2 y_2$

$$20 \times 6 = 30 \times y$$

$\therefore$ $$y = \frac{20 \times 6}{30} = 4$$

Hence, the food will last for 4 days.

11. How many days will 721 persons take to construct the bridge, if 1648 persons can build the same in 21 days?

Solution : Let the required days be y. Since no. of persons decreases, the days will increase. So, the variation is inverse.

No. of persons (x)	1648	721
No. of days (y)	21	y

So, $x_1 y_1 = x_2 y_2$

$$1648 \times 21 = 721 \times y$$

$\therefore$ $$y = \frac{1648 \times 21}{721} = 48$$

Hence, the required days are 48 to construct the bridge.

12. Some spraying work can be done by 17 machines in 120 days. If the work is to be done in 102 days, how many more machines will be required to complete the work?

Solution : Let the required machines be y. Since no. of days decreases, the more machines will be required. So, this is the case of inverse variation.

No. of days (x)	120	102
No. of machines (y)	17	y_2

So, $x_1 y_1 = x_2 y_2$

$$120 \times 17 = 102 \times y_2$$

or $$y_2 = \frac{120 \times 17}{102}$$

$\therefore$ $y_2 = 20$

$\therefore$ So, more machines are required $= 20 - 17$

$= 3$ machines

Exercise-3

1. Alka can do a piece of work alone in 5 days and Priya can do it alone in 10 days and Payal in 15 days. If all three work together, in how many days can they complete the work?

Solution : Work done by Alka in 1 day = $\frac{1}{5}$

Work done by Priya in 1 day = $\frac{1}{10}$

Work done by Payal in 1 day = $\frac{1}{15}$

If they work together, then amount of work done in 1 day = $\frac{1}{5}+\frac{1}{10}+\frac{1}{15}=\frac{11}{30}$

So, the time taken by them to complete the work = $\frac{30}{11}=2\frac{8}{11}$ days.

2. Ravi, Paurav and Suraj together complete a piece of work in 4 days. Ravi can do it alone in 12 days and Paurav alone in 15 days. How many days will Suraj take to complete the work if he works alone ?

Solution : Let the work done by Suraj in a day = $\frac{1}{x}$

Work done by Ravi in a day = $\frac{1}{12}$

Work done by Paurav in a day = $\frac{1}{15}$

Work done by both of them in a day = $\frac{1}{12}+\frac{1}{15}$

$$=\frac{9}{60}=\frac{3}{20}$$

Work done by three of all in a day = $\frac{1}{12}+\frac{1}{15}+\frac{1}{x}$

$$=\frac{3}{20}+\frac{1}{x}$$

Given, work done by three of all in a day = $\frac{1}{4}$

$$\text{then } \frac{3}{20}+\frac{1}{x}=\frac{1}{4}$$

$$\frac{1}{x}=\frac{1}{4}-\frac{3}{20}=\frac{2}{20}=\frac{1}{10}$$

Hence Suraj will take 10 days to complete the work.

3. If A can complete a work in 16 days, B can complete the same work in 18 days and C in 24 days, how long will all three of them will take to complete the work by working together?

Solution : Work done by A in a day = $\frac{1}{16}$

Work done by B in a day $= \frac{1}{18}$

Work done by C in a day $= \frac{1}{24}$

If they work together, the amount of work done in 1 day $= \frac{1}{16} + \frac{1}{18} + \frac{1}{24}$

$$= \frac{23}{144}$$

So, The time taken by them to complete the work $= \frac{144}{23}$ or $6\frac{6}{23}$ days.

4. If tap A can fill $\frac{2}{5}$ of a tank in 12 hours, and tap B can fill $\frac{1}{6}$ of tank in 6 hours, how long will it take to fill the tank completely if both the taps are opened together?

Solution : Tap A can fill the tank in an hour $= \frac{2}{5} \times \frac{1}{2}$

$$= \frac{1}{30}$$

Tap B can fill the tank in an hour $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

If both the taps are opened together, they can fill the tank in an hour $= \frac{1}{30} + \frac{1}{36}$

$$= \frac{11}{180}$$

So, The time taken by both the taps $= \frac{180}{11}$

$$= 16\frac{4}{11} \text{ hours}$$

5. If 49 men can dig a well 28 m deep in 52 days, how many men will be required to dig a well 40 m deep in 35 days?

Solution : Here,

No. of men	Work (depth in m)	No. of days
49	28	52
x	40	28

Then by the formula $\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$

$$\frac{49 \times 52}{28} = \frac{x \times 35}{40}$$

$$\therefore \quad x = \frac{49 \times 52 \times 40}{28 \times 35}$$

$$x = 104$$

So, 104 men will be required to dig the well.

6. *A* can do a piece of work in 10 days. If *A* and *B* work together they can finish it in 6 days. Find

(a) The number of days in which *B* alone will finish the work.

(b) The work left if *B* alone works on it for 3 days only.

Solution : (a) Work done by *A* in a day $= \frac{1}{10}$

Work done by both *A* and *B* in a day $= \frac{1}{6}$

$\therefore$ Work done by *B* in a day $= \frac{1}{6} - \frac{1}{10}$

$= \frac{4}{60}$ or $\frac{1}{15}$

So, 15 days are required in which *B* alone will finish the work.

(b) *B* alone can finish a work in a day $= \frac{1}{15}$ (from part (a))

B alone can finish the work in 3 days $= 3 \times \frac{1}{15}$

$= \frac{3}{15} = \frac{1}{5}$

$\therefore$ The work left $= 1 - \frac{1}{5} = \frac{4}{5}$

7. A bus travels at the speed of 45 km/h. How much distance will it cover in 36 seconds?

Solution : Speed of the bus = 45 km/h $= 45 \times \frac{5}{18}$ m/s $= \frac{25}{2}$ m/s

Time taken = 36 seconds

$\therefore$ distance = speed $\times$ time

$= \frac{25}{2} \times 36$

$= 25 \times 18 = 450$ m

So, the bus will cover 450 m distance in 36 sec.

8. How much time will a 500 m long train take to cross a tree if it is moving at a speed of 72 km/h?

Solution : Speed of the train = 72 km/h

$= 72 \times \frac{5}{18}$ m/s

Distance travelled by the train = 500 m = 20 m/s

$\therefore$ Time taken by the train to cross a tree $= \frac{\text{Distance}}{\text{Speed}}$

$= \frac{500}{20}$

$= 25$ seconds.

Chapter-12 : Understanding Quadrilaterals

Exercise-1

1. Which of the following figures are polygons and which are not? Give reasons for figures which are not polygons.

(a) Not a polygon because its sides intersect each other at points other than the end points.

(b) Yes, a polygon.

(c) Not a polygon as its sides intersect at points other than end points.

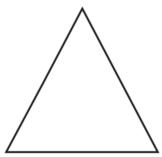
2. Draw two figures for each of the following :

(a) A closed curve that is not a polygon.

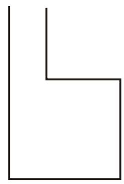
(b) An open curve made up of line segments.

(c) Curves that are neither simple nor closed.

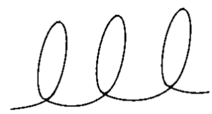
(a)



(b)



(c)



3. Classify the following as simple or complex closed curves :

(a) Complex closed curves

(b) Complex closed curves

(c) Simple closed curves

4. What is the sum of the interior angles of a polygon having,

(a) 5 sides

Substituting $n = 5$ in the formula $\frac{(2n-4)}{n} \times 90°$

Interior angle of a polygon of 5 sides $= \frac{(2 \times 5 - 4) \times 90°}{5}$

$$= \frac{(10-4) \times 90°}{5} = \frac{6 \times 90°}{5} = 6 \times 18°$$

$$= 108°$$

Sum of the interior angles of a polygon $= 5 \times 108°$

$$= 540°$$

(b) 10 sides

Substituting $n = 10$ in the formula $\frac{(2n-4)}{n} \times 90°$

Interior angle of a polygon of 10 sides $= \frac{(2n \times 10 - 4) \times 90°}{10}$

$$= \frac{(20-4) \times 90°}{10} = 16 \times 9 = 144°$$

Sum of the interior angles of a polygon $= 10 \times 144°$

$$= 1440°$$

(c) 9 sides

Substituting $n = 9$ in the formula $\frac{(2n-4)\times 90^\circ}{n}$

Interior angle of a polygon of 9 sides $= \frac{(2\times 9-4)\times 90^\circ}{9}$

$$= \frac{(18-4)\times 90^\circ}{9} = \frac{14\times 90}{9}$$

$$= 14\times 10 = 140^\circ$$

Sum of the interior angles of a polygon $= 9\times 140^\circ$

$$= 1260^\circ$$

5. What is measure of an interior angle of a regular polygon having,

(a) 9 sides

Substituting $n = 9$ in the formula $\frac{(2n-4)\times 90^\circ}{n}$

Interior angle of a polygon of 9 sides

$$= \frac{(2\times 9-4)\times 90^\circ}{9}$$

$$= \frac{(18-4)\times 90^\circ}{9} = \frac{14\times 90}{9}$$

$$= 14\times 10 = 140^\circ$$

(b) 10 sides

Substituting $n = 10$ in the formula $\frac{(2n-4)\times 90^\circ}{n}$

Interior angle of a polygon of 10 sides

$$= \frac{(2\times 10-4)\times 90^\circ}{10} = \frac{(20-4)\times 90^\circ}{10}$$

$$= \frac{16\times 90^\circ}{10} = 16\times 9 = 144^\circ$$

(c) 12 sides

Substituting $n = 12$ in the formula $\frac{(2n-4)\times 90^\circ}{n}$

Interior angle of a polygon of 12 sides $= \frac{(2\times 12-4)\times 90^\circ}{12} = \frac{(24-4)\times 90}{12}$

$$= \frac{20\times 90}{12} = \frac{10\times 90}{6} = 5\times 30 = 150^\circ$$

6. Find the number of sides of a polygon if the sum of the interior angles is :

(a) 720°

Solution : The formula for the total interior angles is

$$(n-2)\times 180^\circ$$

$$(n-2)\times 180 = 720^\circ$$

or $(n-2) = \frac{720}{180}$

$$n-2 = 4$$

or $\quad n = 4 + 2$

$n = 6$

(b) 540°

The formula for the sum of interior angles is $(n-2) \times 180°$

$\therefore \quad (n-2) \times 80 = 540° \quad$ or $\quad (n-2) = \dfrac{540}{180}$

$$(n-2) = \frac{54}{18}$$

$$n - 2 = 3$$

$$n = 3 + 2 = 5$$

(c) 1620°

The formula for the sum of interior angles is $(n-2) \times 180°$

$(n-2) \times 180° = 1620°$

or $\quad (n-2) = \dfrac{1620}{180}$

$$(n-2) = \frac{162}{18}$$

$$(n-2) = 9$$

or $\quad n = 9 + 2 = 11$

7. The angles of pentagon are in the ratio of 1 : 3 : 5 : 4 : 5. Find the angles of the pentagon.

Solution : Ratio of angles $= 1:3:5:4:5$

$$1x + 3x + 5x + 4x + 5x = 540°$$

$$18x = 540$$

or $\quad x = \dfrac{540}{18}$

$$x = 30°$$

Angles of pentagon where $x = 30°$

$$3x = 3 \times 30 = 90°$$

$$5x = 5 \times 30 = 150°$$

$$4x = 4 \times 30 = 120°$$

$$5x = 5 \times 30 = 150°$$

8. What is the minimum interior angle possible for a regular polygon?

Solution : 60°

9. A quadrilateral has three acute angles each measures 80°. What is the measure of fourth angle?

Solution : Three acute angles of a quadrilateral = 80°

Let the measure of fourth angle be x

Sum of interior angles of a quadrilateral = 360°

$$80° + 80° + 80° + x = 360°$$

$$240° + x = 360°$$

or $\quad x = 360° - 240°$

$$x = 120°$$

10. Is it possible to have a regular polygon with an exterior angle of measure 25°?

Solution : No.

11. Find the value of x in each of the following figures :

Solution : (a) Interior angles are $(180-50)=130°$ and $(180-70)=110°$

$\therefore$ Sum of interior angles = 540°

$$40+x+x+130+110=540°$$
$$2x+280=540°$$
$$2x=260$$
or
$$x=\frac{260}{2}$$
$$x=130°$$

Solution : (b) Sum of inteior angles of quadrilateral = 360°

$$120°+50°+80°+x=360°$$
$$250°+x=360°$$
or
$$x=360°-250°$$
$$x=110°$$

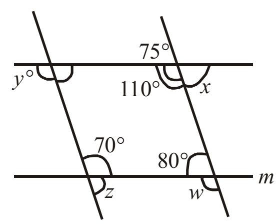
Solution : (c) Sum of interior angles of quadrilateral = 360°

$$45°+25°+x+20°=360°$$
$$90°+x=360°$$
or
$$x=360°-90°$$
$$x=270°$$

Solution : (d) Sum of exterior angles of a quadrilateral = 360°

$$(4x+5)°+x+(x+5)°+(2x-10)°=360°$$
$$8x+10-10=360$$
$$8x=360$$
or
$$x=\frac{360}{8}=45°$$
$$x=45°$$

12. Find the value of angles w, x, y, z in the following figure. Also find their sum.

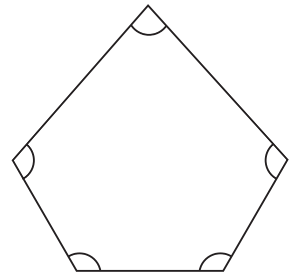


$$110°+x=180°$$
or
$$x=180°-110°$$
$$x=70°$$
$$80+w=180°$$
or
$$w=180°-80°$$
$$=100°$$
$$70°+z=180°$$
or
$$z=180°-70°$$
$$z=110°$$
$$y=180°-100°$$
$$y=80°$$
$$\text{Sum}=w+x+y+z$$
$$=100°+70°+80°+110°$$
$$=360°$$

13. The exterior angles of a pentagon are in the ratio 1 : 2 : 3 : 4 : 5. Find all the interior angles of the pentagon.

Solution : Ratio of the exterior angles of a pentagon $= 1:2:3:4:5$

$$= 1x:2x:3x:4x:5x$$



The sum of exterior angles of a pentagon $= 360°$

$$x + 2x + 3x + 4x + 5x = 360°$$

$$15x = 360°$$

or $$x = \frac{360}{15} = 24$$

$$x = 24°$$

$\therefore$ Exterior angles are $2x = 2 \times 24 = 48°$

$$3x = 3 \times 24 = 72°$$

$$4x = 4 \times 24 = 96°$$

$$5x = 5 \times 24 = 120°$$

$\therefore$ Interior angles are $180° - 24° = 156°$

$$180° - 48° = 132°$$

$$180° - 72° = 108°$$

$$180° - 96° = 84°$$

$$180° - 120° = 60°$$

14. Find the number of sides of a regular polygon whose each exterior angle is 30°.

Exterior angle of a polygon $= 30°$

$$\text{Number of sides in the polygon} = \frac{360°}{x°}$$

$$= \frac{360°}{30°} = 12° \text{ sides}$$

Exercise-2

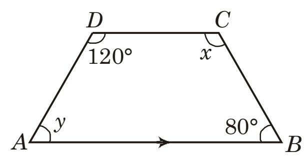
1. $ABCD$ is a trapezium in which $AB \parallel CD$, find x and y.

Solution : $\therefore$ $AB \parallel CD$

$$\angle A + \angle D = 180°$$

$$\angle C + \angle B = 180°$$

$$\angle D = 120°, \quad \angle B = 80°$$



Now $$y + 120° = 180°$$

or $$y = 180° - 120° = 60°$$

$$\angle C + \angle B = 180°$$

$$x + 80° = 180°$$

or $$x = 180 - 80 = 100°$$

$\therefore$ $$y = 60°, \quad x = 100°$$

2. Find the angles x, y, z in the following parallelograms :

(a) In parallelogram

$$10x + 8x = 180°$$

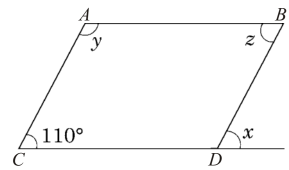
$$18x = 180°$$

$\therefore$ $$x = 100°$$

And $$8x + z = 180°$$

$$8 \times 10 + z = 180°$$
$$80 + z = 180°$$
or $$z = 180° - 80°$$
$$z = 100°$$
Again, $$10x + y = 180°$$
$$10 \times 10 + y = 180°$$
$$100 + y = 180$$
or $$y = 180 - 100$$
$$y = 80°$$

(b) $\angle C + \angle D = 180°$ (Linear pair of angles)



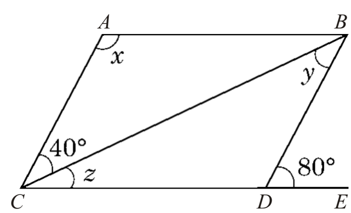
$$110° + \angle D = 180°$$
or $$\angle D = 180° - 110°$$
$$\angle D = 70°$$
$$\angle D + x = 180°$$
$$70° + x = 180°$$
or $$x = 180° - 70° = 110°$$
$y + 110° = 180°$ [Adjacent angles of a parallelogram are supplementary]

or $y = 180° - 110° = 70°$

$y + z = 180°$ [Adjacent angles of a parallelogram are supplementary]

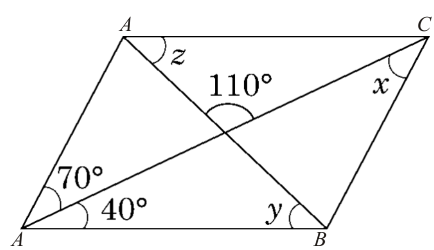
$$70° + z = 180°$$
$$z = 180° - 70° = 110°$$

(c) $$\angle BDC + \angle BDE = 180°$$
$$\angle BDC + 80° = 180°$$
$$\angle BDC = 180° - 80° = 100°$$
$$\angle D + \angle C = 180°$$
[Adjacent angles of a parallelogram are supplementary]



$$100° + (40° + z) = 180°$$
$$140° + z = 180°$$
$$z = 180° - 140° = 40°$$
$$\angle ABC = \angle BCD$$
[Opposite angles]
$$y = 40°$$
$$\angle CAB + \angle ACD$$
[Adjacent angles of a parallelogram are supplementary]
$$x + 80 = 180°$$
$$x = 180° - 80°$$
$$x = 100°$$

(d) $\angle DAC = \angle ACB$ [interior angles]



$$70 = x$$
$$x = 70°$$
$$110° + 40° + y = 180°$$

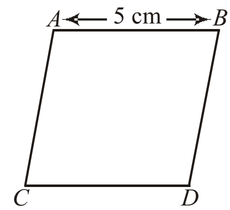
[triangle sum property]

$y = 180 - 150°$

$y = 30°$

$\therefore$ $z = y = 30°$ [Interior angles]

3. In a parallelogram *ABCD*, if $AB = 5$ cm, then $CD = ?$ Give reason.



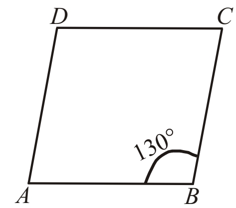
Solution : In a parallelogram *ABCD*

$AB = 5$ cm

$AB \parallel CD$

$AB = CD = 5$ cm [Opposite sides are parallel and equal]

4. (a) In parallelogram *ABCD*, if $\angle B = 130°$, then find $\angle C$.



Solution : In a parallelogam *ABCD*, $\angle B = 130°$

$\angle B = 130°$

$\angle B + \angle C = 180°$

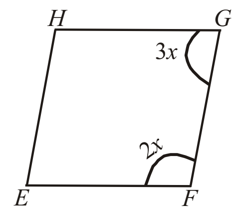
[Adjacent angles of a parallelogram are supplementary]

$130° + \angle C = 180°$

or $\angle C = 180° - 130°$

$\angle C = 50°$

(b) In parallelogram *EFGH*, $\angle F : \angle G = 2:3$, find the measure of $\angle F$ and $\angle G$.



Solution : In parallelogam *EFGH*,

$\angle F : \angle G = 2:3$

Let the adjacent angles be $2x$ and $3x$

$\angle F + \angle G = 180°$

[Adjacent angles of a parallelogram are supplementary]

$5x = 180°$

or $x = \dfrac{180°}{5} = 36°$

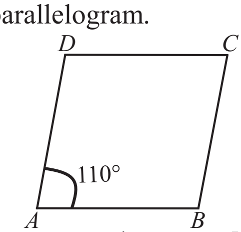
$x = 36°$

Hence, angles are :

$\angle F = 2x = 2 \times 36 = 72°$

$\angle G = 3x = 3 \times 36 = 108°$

5. *ABCD* is a parallelogram and $\angle A = 110°$ Find the other three angles of the parallelogram.



In a parallelogram $\angle A = 110°$

$\angle A + \angle B = 180°$

[Adjacent angles of a parallelogram are supplementary]

$110° + \angle B = 180°$

or $\angle B = 180° - 110° = 70°$

$\angle B + \angle C = 180°$ [Adjacent angles of a parallelogram are supplementary]

$70° + \angle C = 180°$

$\angle C = 180° - 70° = 110°$

$\angle C + \angle D = 180°$ [Adjacent angles of a parallelogram are supplementary]

$110° + \angle D = 180°$

or $\angle D = 180° - 110° = 70°$

6. The perimeter of a parallelogram is 70 cm . If one side is $2 \cdot 5$ times the other, find the sides of the parallelogram.

Solution : Perimeter of a parallelogram = 70 cm

Let shorter side = x cm

Other side = $2 \cdot 5 \times$ shorter side = $(2 \cdot 5 \times x)$ cm

We are given that the total perimeter is 70 cm.

$$2 \times (2 \cdot 5x) + 2 \times x = 70$$
$$5x + 2x = 70$$
$$7x = 70$$
$$x = 10$$

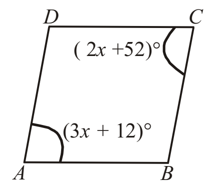
This means the shorter side has length $x = 10$cm, and the longer side has length $2 \cdot 5 \times x = 2 \cdot 5 \times 10 = 25$ cm.

7. Opposite angles of a parallelogram are $(3x + 12)°$ and $(2x + 52)°$. Find all the angles of the parallelogram.

Solution : In a parallelogram $ABCD$, $\angle A = (3x + 12)°$

$$\angle C = (2x + 52)°$$

$\angle A = \angle C$ (Opposite angles of a parallelogram are equal)



$$3x + 12 = 2x + 52$$
$$3x - 2x = 52 - 12$$

or $x = 40°$

$$\angle A = 3x + 12 = 3 \times 40 + 12 = 120 + 12 = 132°$$
$$\angle C = 2x + 52 = 2 \times 40 + 52 = 80 + 52 = 132°$$

$\angle A + \angle B = 180°$ [Adjacent angles of a parallelogram are supplementary]

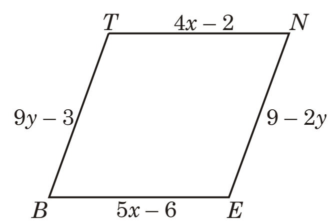
$$132 + \angle B = 180$$
$$\angle B = 180 - 132 = 48°$$

$\angle B = \angle D$ [Opposite angles of a parallelogram]

$$\angle B = \angle D = 48°$$

$\therefore$ $\angle A = 132°$, $\angle C = 132°$, $\angle B = 48°$, $\angle D = 48°$

8. In figure $BENT$ and $RENT$ are parallelograms. Find the value of x and y. (lengths are in cm).



Solution : (a) In parallelogram $BENT$

$$BE = 5x - 6$$
$$TN = 4x - 2$$
$$TB = 9y - 3$$
$$NE = 9 - 2y$$

$BE = TN$ [Opposite sides are parallel and equal]

$$5x - 6 = 4x - 2$$
$$5x - 4x = -2 + 6$$

or $x = 4$

New, $TB = NE$ [Opposite sides are parallel and equal]

$$9y - 3 = 9 - 2y$$
$$9y + 2y = 9 + 3$$
$$11y = 12$$

or $\qquad y = \frac{12}{11} \quad = 1\cdot 2$

9. The angles P, Q, R, S of a quadrilateral $PQRS$ are in the ratio of 1 : 3 : 7 : 9.

(a) Find the measure of each angle.

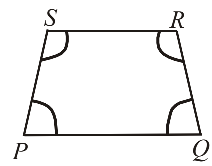
Solution : Let the $\angle P$ be x

Since the angles P, Q, R, S are in the ratio $1:3:7:9$

$\therefore \qquad \angle Q = 3x,\ \angle R = 7x$ and $\angle S = 9x$

We have,

$$x + 3x + 7x + 9x = 360° \qquad \text{[Angle sum property]}$$
$$20x = 360°$$
$$x = \frac{360}{20} = 18°$$



So, $\angle P = x = 18°,\ \angle Q = 3x = 3\times 18 = 54°$

$\angle R = 7x = 7\times 18° = 126°$ and $\angle S = 9\times 18° = 162°$

(b) Is $PQRS$ a trapezium? Why?

$PQRS$ is a trapezium because $PQ \parallel RS$

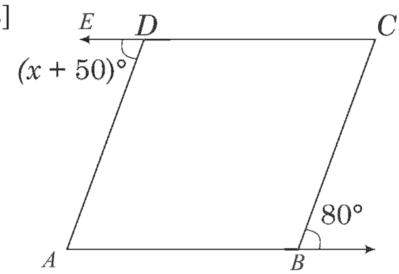
10. $ABCD$ is a parallelogram. What is the value of x?

Solution : In a parallelogram $ABCD$

$$\angle CBA + \angle ABC = 180°$$
$$80° + \angle ABC = 180°$$

or $\qquad \angle ABC = 180° - 80° = 100°$

$$\angle ABC = \angle CDA = 100° \quad \text{[Opposite angles]}$$



Now, $\qquad \angle CDA + \angle EDA = 180°$

$$\angle CDA + x + 50 = 180°$$
$$100 + x + 50 = 180°$$
$$x + 150 = 180$$

or $\qquad x = 180 - 150$

$$x = 30°$$

Exercise-3

1. $PQRS$ is a rhombus. If $PQ = 5$ cm and $OQ = 3$ cm, then what is $PR + SQ$?

Solution : In a rhombus $PQRS$

$$PQ = 5\text{ cm}$$
$$OQ = 3\text{ cm}$$
$$OQ = \frac{1}{2}SQ$$
$$3 = \frac{1}{2}SQ$$
$$SQ = 6\text{ cm}$$

$\angle POQ$ is a right angled triangle. By Pythagoras theorem, we know that

$$PQ^2 = PO^2 + OQ^2$$
$$(5)^2 = PO^2 + (3)^2$$
$$25 = PO^2 + 9$$
$$PO^2 = 25 - 9$$
$$PO^2 = 16$$
$$PO = 4 \text{ cm}$$
$$PO = \frac{1}{2} PR$$
$$4 = \frac{1}{2} PR$$
$$PR = 8 \text{ cm}$$
$$PR + SQ = 8 + 6 = 14 \text{ cm}$$

2. In a rectangle $ABCD$, if $\angle DBC = 60°$, find $\angle DCA$.

Solution : Given $\angle DBC = 60°$

Find $\angle DCB = ?$

In $\Delta\ BOC$

$OB = OC$ [In rectangle diagonals bisect each other]

$\Rightarrow$ $\angle B = \angle C = 60°$

$\therefore$ $\angle BCO = 60°$

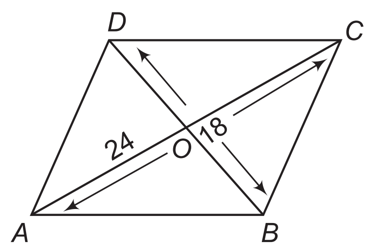
Since, $\angle DCB = 90°$

$\therefore$ $\angle DCA = x = 90° - 60°$

$= 30°$

3. If the diagonals of a rhombus are of length 24 cm and 18 cm, find the length of the side of the rhombus and hence find its perimeter.

Solution : In a rhombus $ABCD$



$AC = 24 \text{ cm}, DB = 18 \text{ cm},$

$\therefore$ $OB = \frac{1}{2} DB$

$= \frac{18}{2} = 9 \text{ cm}$

and $OA = \frac{1}{2} AC = \frac{1}{2} \times 24 = 12 \text{ cm}$

$\angle OAB$ is a right angled triangle. By Pythagoras theorem, we know that

$$AB^2 = AO^2 + OB^2$$
$$AB^2 = (12)^2 + (9)^2$$
$$AB^2 = 144 + 81$$
$$AB^2 = 225$$

$\therefore$ $AB = \sqrt{225}$

$AB = 15$

Hence $AB = BC = CD = DA = 15$ cm

$\therefore$ Perimeter of rhombus $= 4 \times$ side

$= 4 \times 15 = 60$ cm

4. If $ABCD$ is a square, find the measure of $\angle DAC$.

Solution : In square $ABCD$

$$\angle CDA = 90°$$

$$\angle CDA = \angle DAB = 90°$$

$$\angle DAB = \frac{1}{2} CDA = \frac{90}{2} = 45°$$

5. Two adjacent angles of a rhombus are in the ratio of 1 : 5. Find the angles of the rhombus.

Solution : In a rhombus

Two adjacent angles are in the ratio $= 1:5$

Sum of two adjacent angles is equal to 180°

$$x + 5x = 180°$$

$$6x = 180$$

or $$x = \frac{180}{6}$$

$$x = 30$$

One angle $= 30°$

other angle $= 5 \times 30 = 150$

Hence angles are 30°, 150°, 30°, 150°.

6. In rectangle $PQRS$, $PQ = 8$ cm and $PS = 6$ cm. What is the length of SQ ?

Solution : In rectangle $PQRS$

$\angle P = 90° =$ right triangle. By Pythagoras theorem, we know that

$$SQ^2 = SP^2 + PQ^2$$

$$SQ^2 = (6)^2 + (8)^2$$

$$SQ^2 = 36 + 64$$

$$SQ^2 = 100$$

or $$SQ = \sqrt{100}$$

$$SQ = 10 \text{ cm}$$

7. The perimeter of a square is 64 cm. What is the length of the diagonal?

Solution : Perimeter of a square $= 64$

$$4 \times \text{side} = 64$$

$$\text{Side} = \frac{64}{4} = 16 \text{ cm}$$

By letting diagonal equal to D, and using the Pythagorean formula, $a^2 + b^2 = c^2$, we get $(16)^2 + (16)^2 = D^2$

or $$D^2 = 256 + 256$$

or $$D^2 = 512$$

$$D = \sqrt{512} = 16\sqrt{2} \text{ cm}$$

8. In the following squares, find x.

Solution : In ΔPCQ,

(a) $$\angle CPQ = 180° - 120° = 60°$$

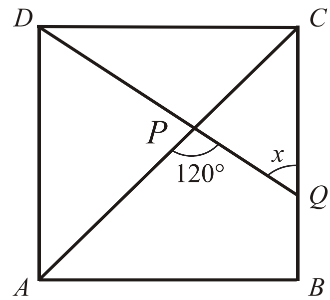
$$\angle PCQ = \frac{1}{2} \times \angle C = \frac{90}{2} = 45°$$

$\therefore$ $$\angle PCQ + \angle CPQ + \angle CQP = 180°$$

$$45° + 60° + x = 180°$$

$$x = 180° - 105°$$

$$x = 75°$$



(b) $$\angle FDA = \frac{1}{2} \times \angle D$$

$$= \frac{1}{2} \times 90° = 45°$$

$$\angle DFE = 180° - 80° = 100°$$

$$\angle DAE = 90°$$

$\therefore$ In Quadilateal $ADFE$,

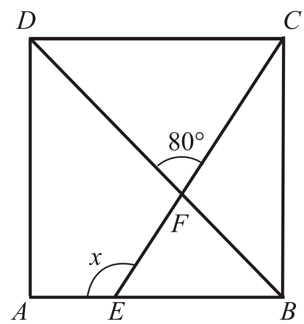
$$\angle DAE + \angle DFE + \angle FEA + FDA = 360°$$

$$90° + 100° + x + 45° = 360°$$

$$235 + x = 360°$$

or $$x = 360° - 235°$$

$\therefore$ $$x = 125°$$



Chapter-13 : Practical Geometry

Execise-1

1. Can you construct a unique quadrilateral $ABCD$ with the given measurements? If not give reason.

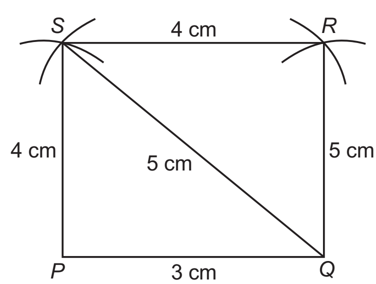
(a) $AB = 7$ cm, $BC = 2$ cm, $AC = 9$ cm, $AD = 6$ cm and $CD = 10$ cm

(b) $AB = 7$ cm, $BC = 5$ cm, $CD = 8$ cm and $AD = 6 \cdot 5$ cm

Solution : (a) Yes.

(b) No. because we have only four elements ; to construct a quadrilateral we should have at least five elements.

2. Construct a quadrilateral $PQRS$, where $PQ = 3$ cm, $QR = 5$ cm, $QS = 5$ cm, $PS = 4$ cm, and $SR = 4$ cm.



Solution : Steps of Construction

(i) Draw a line segment PQ of length 3 cm.

(ii) With P as centre and radius 4 cm, draw an arc.

(iii) With Q as centre and radius 5 cm, draw another arc intersecting arc drawn in step 2, at point S.

(iv) Join PS and QS.

(v) With Q as centre and radius 5 cm, draw an arc on the same side of Q.

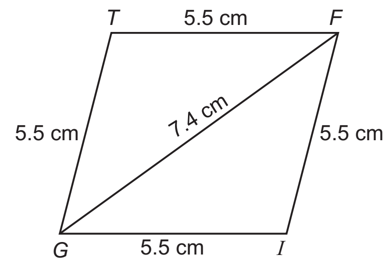
(vi) With S as centre, draw an arc of radius 4 cm intersecting arc drawn in step 5, at point R.

(vii) Join QR and SR .

Thus, $PQRS$ is the required quadrilateral.

3. Construct a rhombus of side $5 \cdot 5$ cm and one of its diagonals of $7 \cdot 4$ cm.

Solution : In a rhombus, we know all sides are equal. Draw a rough sketch of the requied rhombus, say $GIFT$.



Steps of Construction

(i) Draw a line segment GI of length $5 \cdot 5$ cm.

(ii) With G as centre and radius $7 \cdot 4$ cm, draw an arc.

(iii) With I as centre and radius $5 \cdot 5$ cm, draw another arc.

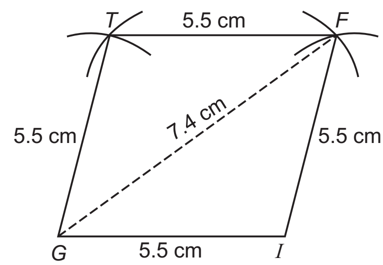
(iv) The two arcs intersect at point F. Join GF and IF.

(v) With G as centre and radius $5 \cdot 5$ cm, draw an arc on the left of F.

(vi) With F as centre and radius $5 \cdot 5$ cm, draw another arc so as to cut the previous arc. The two arcs intersect at a point T.

(vii) Join GT and FT.

Thus, $GIFT$ is the required rhombus.



4. Construct a quadrilateral $PEAR$, where $PE = 10$ cm, $EA = 7 \cdot 5$ cm, $AR = 5$ cm, $RP = AP = 6$ cm.

Solution :Steps of Construction :

(i) Draw a line se gment PE of length 10 cm.

(ii) With P as centre and radius 6 cm, draw an arc.

(iii) With E as centre and radius $7 \cdot 5$ cm, draw another arc intersecting arc drawn in step 2, at point A.

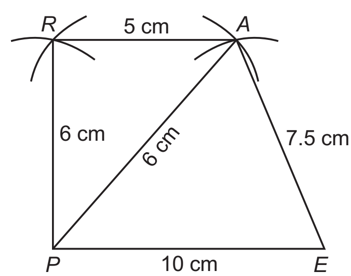
(iv) Join PA and EA.

(v) With P as centre and radius 6 cm draw an arc on the same side of PE where R lies.

(vi) With A as centre and radius 5 cm, draw another arc intersecting arc drawn in step 5 at point R.

(vii) Join PR and AR.

Thus, $PEAR$ is the required quadrilateral.



5. Construct a quadrilateral $SAND$, where $SA = 4 \cdot 5$ cm, $ND = 3 \cdot 2$ cm, $DS = 7 \cdot 5$ cm, $SN = 8$ cm and $AD = 5$ cm.

Solution : Steps of Construction :

(i) Draw a line segment SN of length 8 cm.

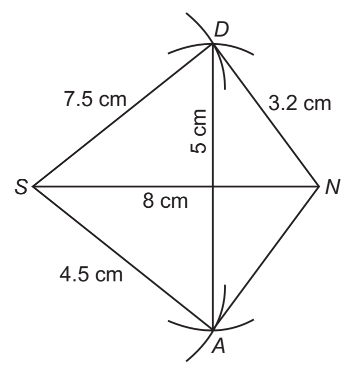
(ii) With S as centre and radius $7 \cdot 5$ cm, draw an arc.

(iii) With N as centre and radius $3 \cdot 2$ cm, draw another arc intersecting arc drawn in step 2, at point D.

(iv) Join SD and ND.

(v) Now, with S as centre and radius $4 \cdot 5$ cm draw an arc on the other side of SN.

(vi) With D as centre and radius 5 cm draw another arc intersecting arc drawn in step 5 at point A.



(vii) Join SA, DA and NA.

Thus, $SAND$ is the required quadrilateral.

6. Construct a quadrilateral $KLMN$, where $KL = 7$ cm, $LM = NK = 6$ cm, $KM = 9$ cm and $LN = 6$ cm.

Steps of Construction :

(i) Draw a line segment $KM = 9$ cm.

(ii) With K as centre and radius 7 cm, draw an arc.

(iii) With M as centre and radius 6 cm, draw another arc intersecting the arc drawn in step 2 at point L.

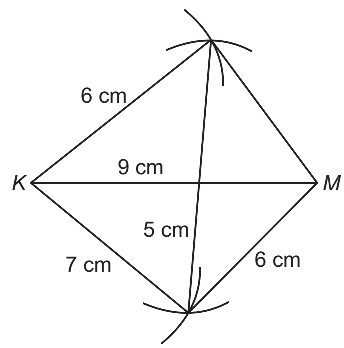
(iv) Join KL and ML.

(v) With K as centre and radius 6 cm, draw an arc on the other side of KM.

(vi) With L as centre and radius 6 cm, draw another arc intersecting arc drawn in step 5 at point N.

(vii) Join KN, MN and NL.

Thus, $KLMN$ is the required Quadrilateral.



7. Construct a square whose diagonal is $6 \cdot 4$ cm.

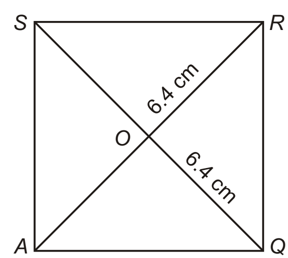
[Hint : Diagonals of a square are equal and find the side of a square then construct.]

In a square, we know that two diagonals are equal. In square $PQRS$, $PR = SQ = 6 \cdot 4$ cm.

As we know diagonals of a square are perpendicular bisectors.

i.e. $OQ = OS = \dfrac{SQ}{2} = \dfrac{6 \cdot 4}{2} = 3 \cdot 2$ cm

Draw a rough figure of square $PQRS$.



Steps of Construction :

(i) Draw line segment $PR = 6 \cdot 4$ cm.

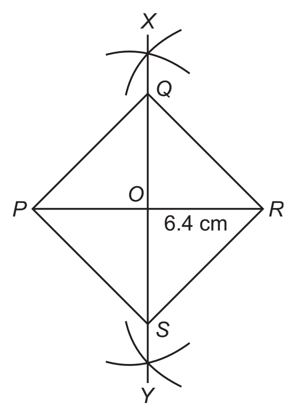
(ii) Construct XY, a perpendicular bisector to PR.

(iii) Let O be the point of intersection of PR and XY.

(iv) With O as centre and radius $3 \cdot 2$ cm, draw arcs to cut OX and OY at points S and Q respectively.

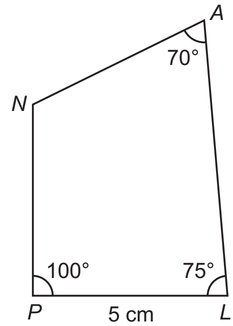
(v) Join PQ, QR, RS and SP

Thus, $PQRS$ is the required square.



Exercise-2

1. Construct a quadrilateral *PLAN*, where $PL = 5$ cm, $LA = 7$ cm, $\angle P = 100°$, $\angle L = 75°$ and $\angle A = 70°$.



Solution : A rough figure of quadrilateral *PLAN* shows that we need to know the measure of angle *NR* to complete our construction. So, we need to first find angle *N* using angle sum property.

i.e. $100 + 75° + 70° + \angle N = 360°$

$245° + \angle N = 360°$

$\angle N = 360° - 245°$

$\angle N = 115°$

Steps of Construction :

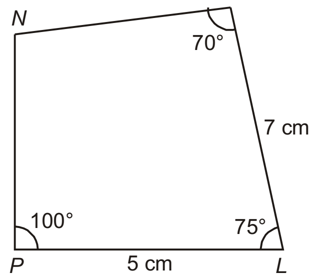
(i) Draw a line segment $PL = 5$ cm.

(ii) Construct angles $\angle NPL = 100°$ at *P* and $\angle PLA = 75°$ at *L*.

(iii) With *L* as centre and radius 7 cm (*LA*), draw an arc at point *A*.

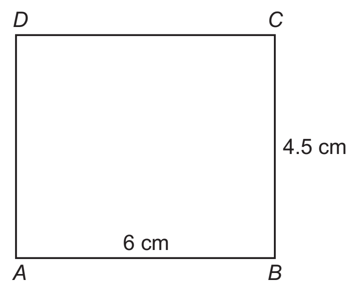
(iv) Now construct and angle $\angle NAL = 70°$ which cuts at point *N*.

(v) Thus, *PLAN* is the required quadrilateral.



2. Construct a rectangle with sides $4 \cdot 5$ cm and 6 cm. Measure its two diagonals.

Solution : Draw a rough sketh of rectangle *ABCD*.



Steps of Construction :

(i) Draw a line segment $AB = 6$ cm.

(ii) Constrcut right angles using compass at points *A* and *B*.

(iii) With *A* and *B* as centres and radius $4 \cdot 5$ cm draw arcs which cut at *C* and *D*.

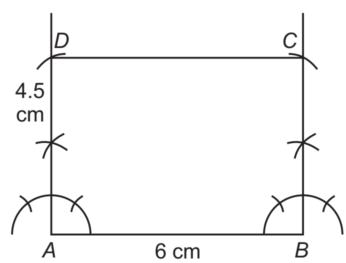
(iv) Join *CD*.

Thus *ABCD* is the required rectangle.

Now, join *AC* and *BD* diagonals.

where $AC = 7 \cdot 5$ cm, $BD = 7 \cdot 5$ cm,

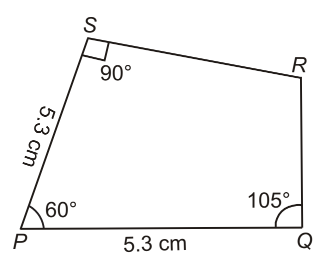
$\therefore$ In rectangle, $AC = BD = 7 \cdot 5$ cm.



3. Construct a quadrilateral *PQRS* where $\angle S = 90°$, $\angle P = 60°$,

$\angle Q = 105°$, $PQ = SP = 5 \cdot 3$ cm

Solution : Draw a rough figure of Quadrilateral *PQRS*.



Steps of Construction :

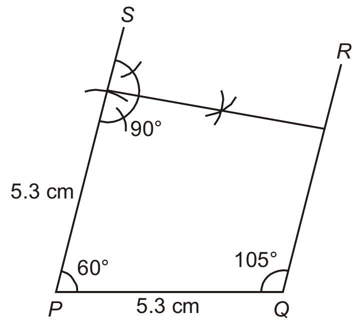
(i) Draw a line segment $PQ = 5 \cdot 3$ cm

(ii) Construct angles $\angle SPQ = 60°$ at P and $\angle PQR = 105°$ at Q.

(iii) With P as centre and radius $5 \cdot 3$ cm draw an arc at point S.

(iv) Now construct right angles $\angle RSP = 90°$ using compass which cuts at R.

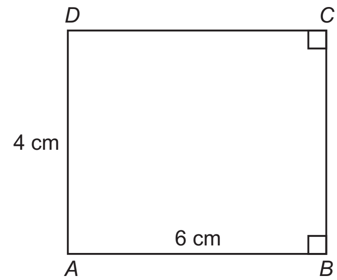
(v) Thus, $PQRS$ is the required quadrilateral.



4. Construct a quadrilateral $ABCD$ where $AB = 6$ cm, $BC = 4$ cm, $AD = 4$ cm, $\angle ABC = \angle BCD = 90°$.

Name this special quadrilateral.

Solution : Draw a rough figure of quadrilateral $ABCD$.



Steps of Construction :

(i) Draw $AB = 6$ cm.

(ii) Construct right angle at point B using compass.

(iii) With B as centre and radius 4 cm draw an arc which cuts at point C.

(iv) Now, at point C construct a right angle.

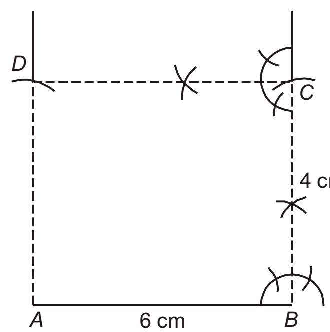
(v) With A as centre and radius 4 cm, draw an arc which cuts the angle drawn in step 4.

Thus, $ABCD$ is the required Quadrilateral.

Here $AB = CD$ and $AD = BC$

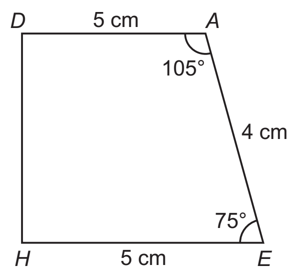
$\angle B = \angle C = 90°$

So, this spacial quadrilateral is **rectangle**.



5. Construct a quadrilateral $HEAD$, where $HE = 5$ cm, $EA = 4$ cm, $AD = 5$ cm, $\angle E = 75°$ and $\angle A = 105°$.

Solution : Draw a rough figure of quadrilateral $HEAD$.



(i) Draw $HE = 5$ cm.

(ii) Construct an angle $\angle AEH = 75°$ at point E.

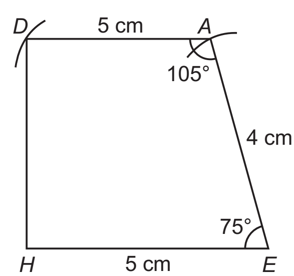
(iii) With E as centre and radius 4 cm, draw an arc which cuts the previous angle at point A.

(iv) Construct an angle $\angle DAE = 105°$ at point A.

(v) With A as centre and radius 5 cm, draw an arc which cuts the previous angle at point D.

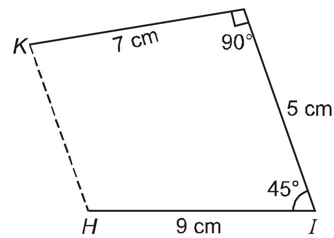
(vi) Join HD.

Thus, $HEAD$ is the required quadrilateral.



6. Construct a quadrilateral $HIJK$ where $HI = 9$ cm, $IJ = 5$ cm, $KJ = 7$ cm, $\angle I = 45°$ and $\angle J = 90°$.

Solution : Draw a rough figure of quadrilateral $HIJK$.



Steps of Construction :

(i) Draw a line $HI = 9$ cm.

(ii) Using compass construct an angle of 45° at point I.

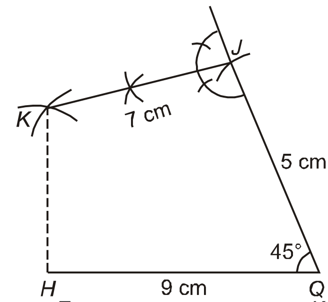
(iii) With I as centre and radius 5 cm, draw an arc, which cuts the previous angle at point J.

(iv) Using compass construct a right angle at point J.

(v) With J as centre and radius 7 cm, draw an arc which cuts the previous angle at point K.

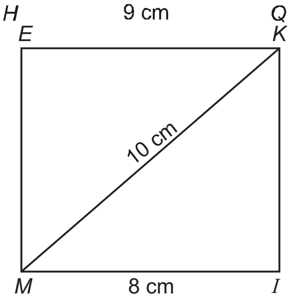
(vi) Join HK.

Thus, $HIJK$ is the required quadrilateral.



7. Construct a rectangle $MIKE$, where $MI = 8$ cm and $MK = 10$ cm.

Solution : Draw a rough figure of rectangle $MIKE$.



Steps of Construction :

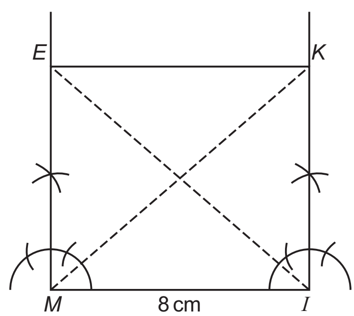
(i) Draw a line $MI = 8$ cm.

(ii) Construct right angles at points M and I.

(iii) With M as centre and radius 10 cm (MK), draw an arc at K. Also taking I as centre and radius 10 cm ($MK = IE$), draw an arc at E.

(iv) Join EK.

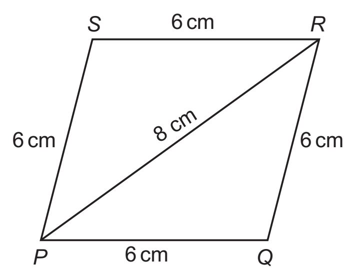
Thus, $MIKE$ is the required rectangle.



8. Construct a rhombus $PQRS$, where $PQ = 6$ cm and $PR = 8$ cm. What is the measure of QS?

Solution : In a rhombus, we know that all sides are equal.

Draw a rough sketch of the required rhombus $PQRS$.



Steps of Construction :

(i) Draw a line segment $PQ = 6$ cm.

(ii) With P as centre and radius 8 cm, draw an arc.

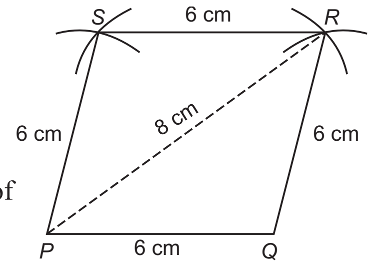
(iii) With Q as centre and radius 6 cm, draw another arc.

(iv) The two arcs intersect at point R. Join PR and RQ.

(v) With P as centre and radius 6 cm, draw an arc on the left of R.

(vi) With R as centre and radius 6 cm, draw another arc so as to cut the previous arc. The two arcs intersect at a point S.

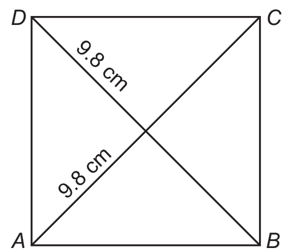
(vii) Join PS and RS.



9. Construct a square whose diagonal is equal to $9 \cdot 8$ cm. What is the measure of its side?

Solution : In a square we know two diagonals are equal.

In square $ABCD$, $AC = BD = 9 \cdot 8$ cm, as we know diagonals of a square are perpendicular bisectors.



$$i.e.\ OD = OB = \frac{AC}{2}$$

$$= \frac{9 \cdot 8}{2}$$

$$= 4 \cdot 9 \text{ cm}$$

Draw a rough sketh of square $ABCD$.

Steps of construction :

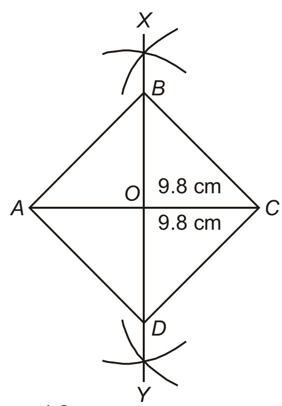
(i) Draw a line segment $AC = 9 \cdot 8$ cm.

(ii) Construct XY a perpendicular bisector to AC

(iii) Let O be the point of intersection of AC and BD.

(iv) With O as centre and radius $4 \cdot 9$ cm draw arcs to cut OX and OY at points B and D.

(v) Join AB, BC, CD and DA. Thus, $ABCD$ is the required square.



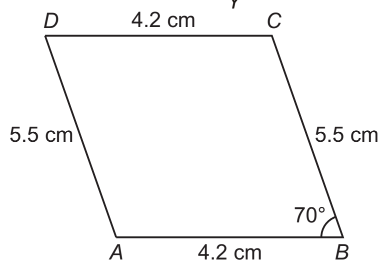
Now, side of square $= \sqrt{(4 \cdot 9)^2 + (4 \cdot 9)^2}$

$= \sqrt{48 \cdot 02}$

$= 6 \cdot 92$ cm (approx.)

10. Construct a parallelogram $ABCD$ in which $AB = 4 \cdot 2$ cm, $BC = 5 \cdot 5$ cm and $\angle B = 70°$.

Solution : In a parallelogram, we know that opposite sides are equal. Draw a rough sketch of parallelogram $ABCD$.



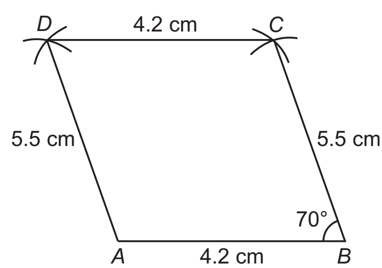
Steps of Construction :

(i) Draw a line segment $AB = 4 \cdot 2$ cm.

(ii) Construct an angle of 70° at point B.

(iii) With B as centre draw an arc of radius $5 \cdot 5$ cm which cuts at C.

(iv) Now, with C as centre and radius $4 \cdot 2$ cm ($AB = CD$) draw an arc. Also with A as centre and radius $5 \cdot 5$ cm ($AD = BC$) draw another arc.

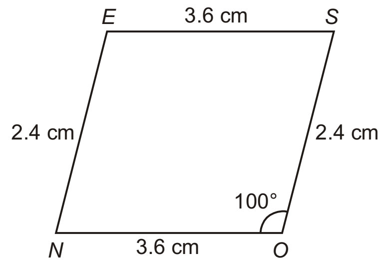


(v) The two arcs intersect at point D.

Thus, $ABCD$ is the required parallelogram.

11. Construct a parallelogram with sides of lengths $3 \cdot 6$ cm and $2 \cdot 4$ cm and one of its angles $100°$.

Measure the shorter diagonal.



Solution : In a parallelogram, we know that opposite sides are equal. Draw a rough figure of parallelogram, say $NOSE$.

Steps of Construction :

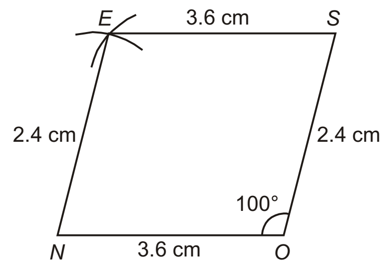
(i) Draw a line $NO = 3 \cdot 6$ cm.

(ii) Construct an angle of $100°$ at point O.

(iii) With O as centre and radius $2 \cdot 4$ cm, draw an arc which cuts the previous angle at point S.

(iv) With S as centre and radius $3 \cdot 6$ cm, draw an arc. Also with N as centre and radius $2 \cdot 4$ cm, draw another arc.

(v) The two arcs intersect at point E.

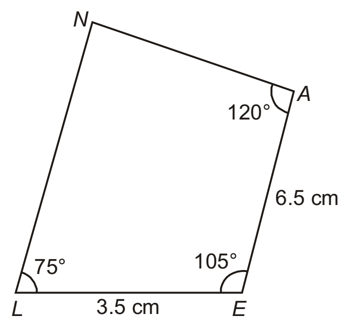


Thus, $NOSE$ is the required parallelogram.

$\therefore$ Shorter diagonal $EO = 3 \cdot 9$ cm.

12. Construct a quadrilateral $LEAN$, where $LE = 3 \cdot 5$ cm, $EA = 6 \cdot 5$ cm, $\angle L = 75°$, $\angle E = 105°$ and $\angle A = 120°$.

Solution : Draw a rough sketch of quadrilateral $LEAN$.



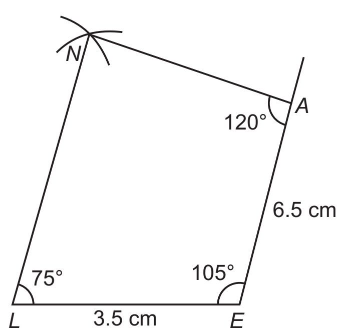
Steps of Construction :

(i) Draw a line LE $= 3 \cdot 5$ cm.

(ii) Construct the angles $\angle NLE = 75°$ at point L and $\angle LEA = 105°$ at point E.

(iii) With E as centre and radius $6 \cdot 5$ cm, draw an arc which cuts the previous angle $\angle E$ at point A.

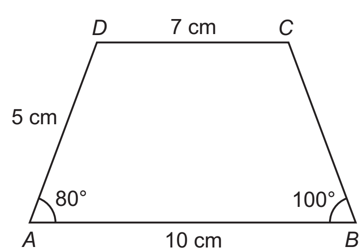
(iv) Now, construct an angle of $120°$ at point A which cuts the previous angle $\angle L$, at point N.



Thus, $LEAN$ is the required quadrilateral.

13. Construct a trapezium $ABCD$, where $AB \parallel DC$, $AB = 10$ cm, $DC = 7$ cm, $AD = 5$ cm and $\angle A = 80°$.

Solution : In a trapezium, we know that sum of adjacent angles is $180°$.



$$i.e.\quad \angle A + \angle B = 180°$$
$$80° + \angle B = 180°$$
$$\angle B = 180° - 80°$$
$$\angle B = 100°$$

Draw a rough figure of Trapezium $ABCD$

Step of Construction

(i) Draw a line segment $AB = 10$ cm.

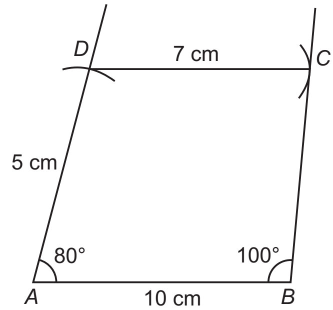
(ii) Construct an angle $\angle DAB = 80°$ at point A

(iii) With A as centre and radius 5 cm, draw an arc which cuts the previous angle at point D.

(iv) Construct an angle $\angle ABC = 100°$ at point B.

(v) With D as centre draw an arc of radius 7 cm, which cuts the angle drawn in step 2 at point C.

Thus, $ABCD$ is the required trapezium.



Chapter-14 : Visualizing Solid Shapes

Exercie-1

1. Name the solids that have :

(a) 4 faces

(b) 5 faces and 5 vertices

(c) 2 pentagonal faces and 5 rectangular faces.

Solution : (a) Tetrahedron (b) Square pyramid

(c) Pentagonal prism

2. How many faces, edges and vertices does each of the following solid figures have?

(a) Tetrahedron (b) Rectangular pyramid

(c) Octahedron (d) A decagonal pyramid

Solution : (a) 4 faces, 6 edges and 4 vertices.

(b) 5 faces, 8 edges and 5 vertices.

(c) 8 faces, 12 edges and 6 vertices.

(d) 11 faces, 20 edges and 11 vertices.

3. How many faces does a polyhedron have if it has 45 edges and 30 vertices?

Solution : Edges $(E) = 45$

Vertices $(V) = 30$

By Euler's formula :

$$F + V = E + 2$$

$$F + 30 = 45 + 2$$

or $$F = 45 + 2 - 30$$

$$F = 17$$

So, no. of faces in the polyhedron is 17.

4. How many edges does a polyhedron have if it has 26 vertices and 15 faces?

Solution : Vertices $(V) = 26$

Faces $(F) = 15$

by Euler's formula :

$$F + V = E + 2$$

$$15 + 26 = E + 2$$

or $E = 15 + 26 - 2$

$\therefore$ $E = 39$

Thus, no. of edges in the polyhedron is 39.

5. Find the number of edges in a pyramid with hexagonal base.

Solution : In a pyramid with hexagonal base,

No. of vertices $(V) = 8$

No. of faces $(F) = 6$

By Euler's formula :

$$F + V = E + 2$$

$$6 + 8 = E + 2$$

or $E = 6 + 8 - 2$

$$E = 12$$

So, no. of edges in the polyhedron is 12.

6. Fill in the blanks using Euler's formula and also name the solids formed.

Solution : (a) $V = 5$

$$E = 8$$

by Euler's formula $F + V = E + 2$

$$F + 5 = 8 + 2$$

or $F = 8 + 2 - 5$

$$F = 5$$

and, the name of solid is square pyramid.

(b) $F = 6$

$$E = 12$$

By Euler's formula, $F + V = E + 2$

$$6 + V = 12 + 2$$

or $V = 12 + 2 - 6$

$$V = 8$$

and, the name of solid is cube or cuboid.

(c) $F = 8$

$$V = 6$$

by Euler's formula.

$$F + V = E + 2$$

$$8 + 6 = E + 2$$

or $E = 8 + 6 - 2$

$$E = 12$$

and, the name of solid is octahedron.

Exercise 2

Do yourself.

Chapter-15 : Mensuration

Exerise-1

1. The side of a square exceeds the side of another square by 5 cm and the sum of the areas of two squares is $325\,cm^2$. Find the side of each square.

Solution : Let the side of a square be x cm, then the side of another square be $(x+5)$ cm.

Now, according to the question,

$$x^2 + (x+5)^2 = 325$$
$$x^2 + x^2 + 25 + 10x = 325$$
$$2x^2 + 10x - 300 = 0$$
$$x^2 + 5x - 150 = 0$$
$$x^2 + (15-10)x - 150 = 0$$
$$x^2 + 15x - 10x - 150 = 0$$
$$x(x+15) - 10(x+15) = 0$$
$$x(x+15) - 10(x+15) = 0$$
$$(x+15)(x-10) = 0$$
$$x + 15 \neq 0$$

So, $x - 10 = 0$

or $x = 10$

∴ Side of a square $= x = 10$ cm

Side of another square $= x + 5$

$= 10 + 5 = 15$ cm

2. A rectangular floor of dimensions $4\cdot2\text{ m} \times 5\cdot4\text{ m}$ is to be tiled with right triangular marble pieces, with base 40 cm and height 30 cm. How many marble pieces will be required to cover the floor ?

Solution : No. of required marble pieces $= \dfrac{\text{Area of rectangular floor}}{\text{Area of right triangular marble piece}}$

$$= \frac{4\cdot2\text{ m} \times 5\cdot4\text{ m}}{\frac{1}{2} \times 40\text{ cm} \times 30\text{ cm}}$$
$$= \frac{420\text{ cm} \times 540\text{ cm}}{20\text{ cm} \times 30\text{ cm}}$$
$$= \frac{42 \times 54}{6}$$
$$= 42 \times 9$$
$$= 378$$

3. Mrs. Rathor has a square plot with the measurement as shown in the figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house, if cost of developing a garden is ₹ 55 per square metre.

Solution : In the outer rectangle,

Length = 25 m

Breadth = 25 m

$\therefore$ Area of outer rectangle = $l \times b$

$= 25 \times 25 \text{ m}^2$

$= 625 \text{ m}^2$

In the inner rectangle,

Length = 20 m

Breadth = 15 m

$\therefore$ Area of inner rectangle = $l \times b$

$= 20 \times 15$

$= 300 \text{ m}^2$

Now, Area of garden = Area of outer rectangle – Area of inner rectangle

$= (625 - 300) \text{ m}^2$

$= 325 \text{ m}^2$

Cost of developing the garden ₹ 55 per m^2

= ₹ 325×55

= ₹ 17875

4. Two cross roads each 2 m wide run at right angles through the centre of a rectangular park of length 72 m and breadth 48 m such that each is parallel to one of the sides of the rectangle. Find the area of the roads. Also find the area of the remaining portion of the park.

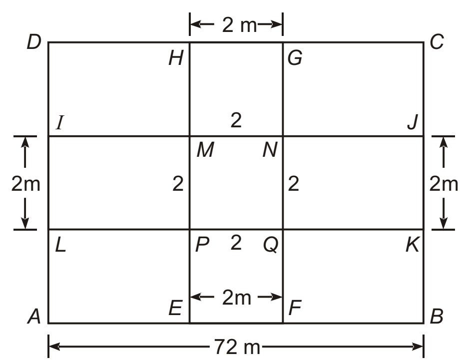
Solution : Length of park (AB) = 72 m

Breadth of park (BC) = 48 m

Area of rectangle $ABCD$ = $AB \times BC$

$= 72 \times 48$

$= 3456 \text{ m}^2$



Length of first road (LK) = 72 m

Breadth of first road (KJ) = 2 m

$\therefore$ Area of first road = $LK \times KJ$

$= 72 \times 2 = 144 \text{ m}^2$

And length of second road (HE) = 48 m

Breadth of second road (HG) = 2 m

Area of second road = $HE \times HG$

$= 48 \times 2 = 96 \text{ m}^2$

$\therefore$ Area of $MNOP$ (square) = $s \times s$

$= 2 \times 2 = 4 \text{ m}^2$

$\therefore$ Area of crossing roads = $(144 + 96 - 4)$

$= 236 \text{ m}^2$

Now, Area of the remaining portion of the park = (Area of $ABCD$ – Area of crossing roads)

$= (3456 - 236) \text{ m}^2$

$= 3220 \text{ m}^2$

5. A field is in the form of a triangle. If the length of the base of the field is 200 m and its area is 17,500 m ^{2}, find the altitude of the field.

Solution : In the right trianglular field,

$$\text{Length of base} = 200 \text{ m}^2$$

$$\text{Area of field} = 17500 \text{ m}^2$$

$$\therefore \quad \text{Area of right triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$17500 = \frac{1}{2} \times 200 \times \text{ altitude}$$

$$\text{Altitude} = \frac{17500 \times 2}{200}$$

$$= 175 \text{ m}$$

6. A park is in the shape of a circle and a person walking on the boundary covers a distance of 440 m in one round. Find the diameter of the park and also the cost of covering it with gravel at the rate of ₹ 225 per m ^{2}.

Solution : Total distance covered by a person in a circular park = 440 m

$$\therefore \quad \text{Circumference of circle} = 440$$

$$2\pi r = 440$$

$$2 \times \frac{22}{7} \times r = 440$$

$$r = \frac{440 \times 7}{2 \times 22}$$

$$r = 70 \text{ m}$$

$$\therefore \quad \text{Diameter of the park} = 2 \times r$$

$$= 2 \times 70 \text{ m}$$

$$= 140 \text{ m}$$

$$\therefore \quad \text{Area of circular park} = \pi r^2$$

$$= \frac{22}{7} \times 70 \times 70$$

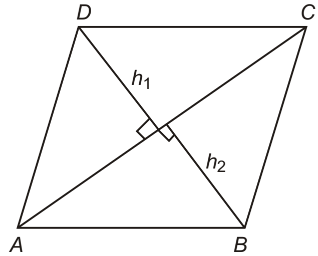
$$= 15400 \text{ m}^2$$

∴ Cost of covering the park with gravel at the rate of ₹ 225 per m ^{2}

$$= ₹\, 15400 \times 225$$

$$= ₹\, 34{,}65{,}000$$

7. A field in the shape of a quadrilateral has its diagonal as 28 m and the perpendicular distances of the other two vertices from the diagonal are $32 \cdot 5$ m and 22 m. Find the area of the field.



Solution : Length of diagonal $AC = 28$ m

$$\text{Heights, } h_1 = 32 \cdot 5 \text{ m}$$

$$h_2 = 22 \text{ m}$$

$$\therefore \quad \text{Area of field} = \frac{1}{2} \times AC \times (h_1 + h_2)$$

$$= \frac{1}{2} \times 28 \times (32 \cdot 5 + 22)$$

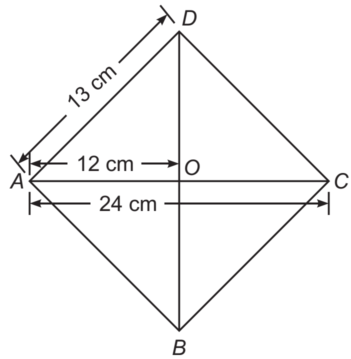
$$= 14 \times 54 \cdot 5$$

$$= 763 \cdot 0 \text{ m}^2$$

8. Find the area of rhombus having side equal to 13 cm and one of whose diagonals is 24 cm.

Solution : Side of rhombus = 13 cm

One of diagonals = 24 cm



In right angled $\Delta\ AOD$,

$$(AD)^2 = (OA)^2 + (OD)^2$$

$$(13)^2 = (12)^2 + (OD)^2$$

$$169 = 144 + (OD)^2$$

$$(OD)^2 = 169 - 144$$

$$= 25$$

$\therefore$ $OD = 5$ cm

So, the length of other diagonal is 10 cm

$$\text{Now, Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 24 \times 10 \text{ m}^2$$

$$= 120 \text{ m}^2$$

9. A cycle wheel has a radius of 28 cm. What distance will it travel in one revolution and in 24 revolutions ?

Solution : Radius of wheel = 28 cm

Distance covered in one revolution $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 28$$

$$= 2 \times 22 \times 4$$

$$= 176 \text{ cm}$$

and, distance covered in 24 revolutions $= 176 \times 24$

$$= 4224 \text{ cm}$$

10. A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2.

Solution : $$\text{No. of tiles} = \frac{\text{Area of floor}}{\text{Area of a flooring tile}}$$

$$= \frac{1080 \text{ m}^2}{24 \text{ cm} \times 10 \text{ cm}}$$

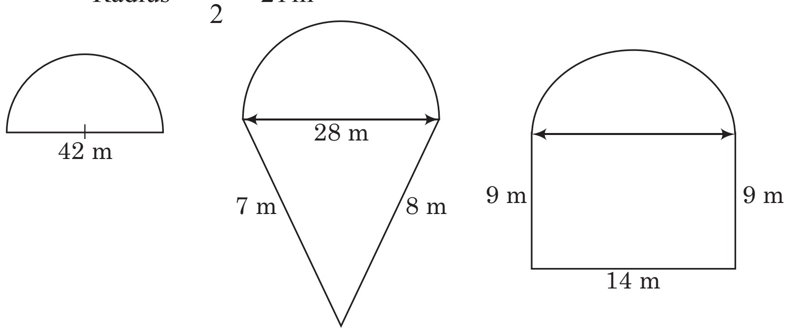
$$= \frac{1080 \times 100 \times 100 \text{ cm}^2}{24 \times 10 \text{ cm}^2}$$

$$= 45000$$

11. The following are the shapes of different flower beds in a garden. If all of them are to be fenced using wire and the cost of wire is ₹ 7 per m, find the total cost of wire. Which flower bed requires the longest fence?

Solution : In first figure,

$$\text{Radius} = \frac{42}{2} = 21\,\text{m}$$



$$\text{Length of wire} = \frac{2\pi r}{2}$$

$$= \frac{2}{2} \times \frac{22}{7} \times 21$$

$$= 66\,\text{m}$$

$$\text{Cost of wire to be fenced} = ₹\, 66 \times 7$$

$$= ₹\, 462$$

In second figure,

$$\text{Length of wire} = \pi r + (28 + 7 + 8)$$

$$= \frac{22}{7} \times \frac{28}{2} + 43$$

$$= 44 + 43$$

$$= 87\,\text{m}$$

$$\text{Cost of wire to be fenced} = ₹\, 87 \times 7$$

$$= ₹\, 609$$

In third figure,

$$\text{Length of wire} = \pi r + 2\,(l + b)$$

$$= \frac{22}{7} \times \frac{14}{2} + 2\,(14 + 9)$$

$$= 22 + 2 \times 23$$

$$= 22 + 46$$

$$= 68\,\text{m}$$

$$\text{Cost of wire to be fenced} = ₹\, 68 \times 7$$

$$= ₹\, 476$$

$\therefore$ Total cost of wire = ₹ (462 + 609 + 476)

$$= ₹\, 1547$$

$\therefore$ Semi circular flower bed (first figure) requires the longest fence.

Exercise-2

1. The area of a trapezium is $984\ cm^2$. If lengths of the parallel sides are $31\cdot5$ cm and $50\cdot5$ cm, find the distance between them.

Solution : Area of trapezium $= 984\ cm^2$

Lengths of parallel sides,

$$a = 31\cdot5 \text{ cm}$$
$$b = 50\cdot5 \text{ cm}$$

$\therefore$ $$\text{Area of trapezium} = \frac{1}{2}\times h\times(a+b)$$
$$984 = \frac{1}{2}\times h\times(31\cdot5+50\cdot5)$$
$$984 = \frac{1}{2}\times h\times 82\cdot0$$
$$h = \frac{984\times2}{82}$$
$$h = 24 \text{ cm}$$

Thus, the distance between parallel sides is 24 cm.

2. The area of a trapezium is $105\ cm^2$ and its height is 7 cm. If one of the parallel sides is longer than the other by 6 cm, find the two parallel sides.

Solution : Area of trapezium $= 105\ cm^2$

Height $(h) = 7$ cm

Let one of parallel sides $(a) = x$ cm

Other side $(b) = (x+6)$ cm

$\therefore$ $$\text{Area of trapezium} = \frac{1}{2}\times h\times(a+b)$$
$$105 = \frac{1}{2}\times7\times(x+x+6)$$

or $$105 = \frac{1}{2}\times7\times(2x+6)$$
$$2x+6 = \frac{105\times2}{7}$$
$$2x+6 = 30$$
$$2x = 30-6$$
$$2x = 24$$

or $$x = 12 \text{ cm}$$

$\therefore$ One side $= x = 12$ cm

Other side $= x+6 = 12+6 = 18$ cm

3. The ratio of the length of parallel sides of a trapezium is 2 : 3. The distance between them is 16 cm. If the area of the trapezium is $320\ cm^2$, find the length of the parallel sides.

Solution : Let lengths of parallel sides of a trapezium, $a = 2x$

$$b = 3x$$

and height $(h) = 16$ cm

$$\text{Area} = 320 \text{ cm}^2$$

$$\text{Area of trapezium} = \frac{1}{2} \times h \times (a+b)$$

$$320 = \frac{1}{2} \times 16 \times (2x + x)$$

$$320 = 8 \times 5x$$

$$320 = 40x$$

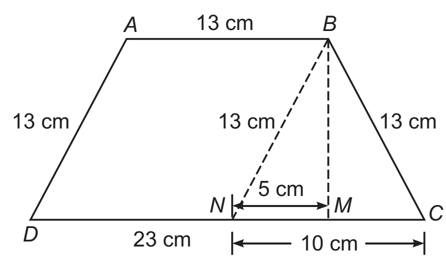
$\therefore$ $$x = \frac{320}{40} = 8$$

So, $a = 2x = 2 \times 8 = 16$ cm

$b = 3x = 3 \times 8 = 24$ cm

4. In a trapezium $ABCD$, $AB = AD = BC = 13$ cm and $CD = 23$ cm. Find the area of the trapezium.

Solution : In right angled $\Delta\, BNM$,



$$(BN)^2 = (BM)^2 + (NM)^2$$

$$(13)^2 = (BM)^2 + (5)^2$$

$$169 = (BM)^2 + 25$$

$$(BM)^2 = 169 - 25$$

$$= 144$$

or $$BM = 12$$

$\therefore$ $$BM = \text{height} = 12 \text{ cm}$$

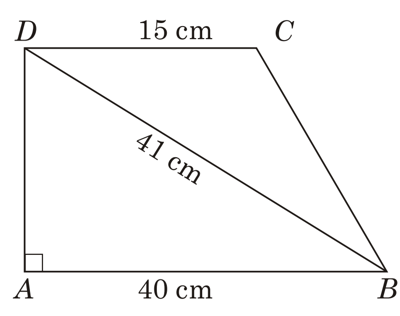
$\therefore$ $$\text{Area of trapezium} = \frac{1}{2} \times h \times (a+b)$$

$$= \frac{1}{2} \times 12 \times (13 + 23)$$

$$= 6 \times 36 = 216 \text{ cm}^2$$

5. From the adjoining figure calculate :

(a) The length AD



Solution : In $\Delta\, DAB$ Length of AD,

$$(AD)^2 = (BD)^2 - (AB)^2$$

$$= (41)^2 - (40)^2$$

$$= 1681 - 1600$$

$$(AD)^2 = 81$$

$\therefore$ $$AD = 9 \text{ cm}$$

(b) The area of trapezium $ABCD$

Solution : Area of trapezium $ABCD = \frac{1}{2} \times h \times (a+b)$

$$= \frac{1}{2} \times 9 \times (15 + 40)$$

$$= \frac{1}{2} \times 9 \times 55$$

$$= 247 \cdot 5 \text{ cm}^2$$

(c) The area of triangle *BCD*

Solution : Area of triangle *BCD* = (Area of trapezium − Area of Δ *ADB*)

$$= \left[\frac{1}{2} \times h \times (a + b) - \frac{1}{2} \times \text{base} \times \text{height}\right]$$

$$= \left[\frac{1}{2} \times 9 \times (15 + 40) - \frac{1}{2} \times AB \times AD\right]$$

$$= \left[\frac{1}{2} \times 9 \times 55 - \frac{1}{2} \times 40 \times 9\right]$$

$$= [247 \cdot 5 - 180]$$

$$= 67 \cdot 5 \text{ m}^2$$

6. Find the area of a trapezium whose parallel sides are 80 dm and 6 m and altitude is 40 dm.
[**Hint :** 10 dm = 1 m]

Solution : Sides, $a = 80 \text{ dm} = 8 \text{ m}$

$b = 6 \text{ m}$

Altitude, $h = 40 \text{ dm} = 4 \text{ m}$

$\therefore$ Area of trapezium $= \frac{1}{2} \times h \times (a + b)$

$$= \frac{1}{2} \times 4 \times (8 + 6) \text{ m}^2$$

$$= 2 \times 14 = 28 \text{ m}^2$$

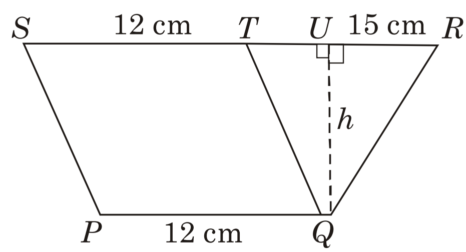
7. In the given figure, *PQTS* is a parallelogram in which $PQ = TS = 12$ cm and the area of Δ *QRT* is 75 cm^2. If $TR = 15$ cm, find the area of trapezium *PQRS*.

Solution : In Δ *QRT*,

Area of Δ $QRT = \frac{1}{2} \times b \times h$

$$75 = \frac{1}{2} \times 15 \times h$$

$$h = 10 \text{ cm}$$



$\therefore$ Area of trapezium $PQRS = \frac{1}{2} \times h \times (a + b)$

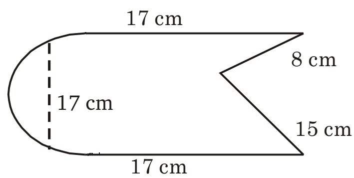
$$= \frac{1}{2} \times 10 \times (12 + 27)$$

$$= 5 \times 39$$

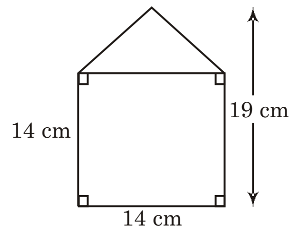
$$= 195 \text{ cm}^2$$

Exercise-3

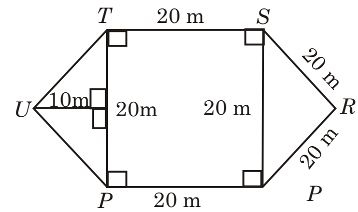
1. Find the area enclosed by each of the following figures :



(a)

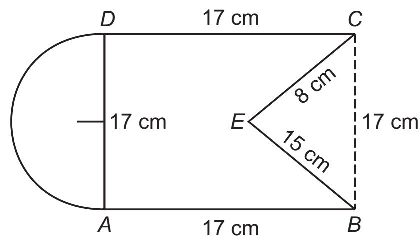


(b)



(c)

Solution : (a) $AB = 17$ cm, $BC = 17$ cm



$\therefore$ Area of $ABCD = 17 \times 17 = 289\text{ cm}^2$

$\therefore$ Area of semicircle $= \dfrac{\pi r^2}{2} = \dfrac{22}{7} \times \dfrac{17}{2} \times \dfrac{17}{2} \times \dfrac{1}{2}$

$= 113 \cdot 535\text{ cm}^2$

$\therefore$ Area of $\Lambda\, BEC = \dfrac{1}{2} \times b \times h$

$= \dfrac{1}{2} \times 15 \times 8 = 60\text{ cm}^2$

$\therefore$ Area of enclosed figure $= (289 + 113 \cdot 535 - 60)\text{ cm}^2$

$= 342 \cdot 535 \approx 342 \cdot 54\text{ cm}^2$

(b) See Figure (b).

Area enclosed by figure = Area of Δ + Area of square

$= \dfrac{1}{2} \times b \times h + \text{side} \times \text{side}$

[$\because$ h of triangle $= 19 - 14 = 5$ cm]

$= \dfrac{1}{2} \times 14 \times 5 + 14 \times 14$

$= 35 + 196$

$= 231\text{ cm}^2$

(c) Area of enclosed by figure (c),

= Area of $\Delta\, TUP$ + Area of $\Delta\, SQR$ + Area of square PQST

$= \dfrac{1}{2} \times b \times h_1 + \dfrac{1}{2} \times b \times h_2 + \text{side} \times \text{side}$

$= \dfrac{1}{2} \times 20 \times 10 + \dfrac{1}{2} \times 20 \times 10\sqrt{3} + 20 \times 20$

$= 100 + 100\sqrt{3} + 400$

$= 100 + 173 \cdot 2 + 400$

$= 673 \cdot 2\ \text{cm}^2$

$[h^2 \text{ of } SQR = (20)^2 - (10)^2 = 400 - 100 \quad h^2 = 300 h = 10\sqrt{3}\ \text{cm}]$

2. The area of a rhombus is $216\ \text{cm}^2$ and one of its diagonals is 18 cm. Find the other.

Solution : Area of rhombus $= 216\ \text{cm}^2$

Diagonal $d_1 = 18\ \text{cm}$

$\therefore$ Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$$216 = \frac{1}{2} \times 18 \times d_2$$

or $$d_2 = \frac{216 \times 2}{18}$$

$$= 24\ \text{cm}$$

3. The area of a trapezium is $276\ \text{cm}^2$ and its height is 24 cm. If one of the parallel sides is longer than the other by 7 cm, find the length of both the parallel lines.

Solution : Let one of the parallel sides be x cm

Then, other side = $(x + 7)$ cm

Area of trapezium $= 276\ \text{cm}^2$

$\therefore$ Area of trapezium $= \frac{1}{2} \times h \times (a + b)$

$$276 = \frac{1}{2} \times 24 \times (x + x + 7)$$

or $$276 = 12 \times (2x + 7)$$

$$\frac{276}{12} = 2x + 7$$

$$23 = 2x + 7$$

$$23 - 7 = 2x$$

$$2x = 16$$

or $$x = \frac{16}{2} = 8$$

$$x = 8\ \text{cm}$$

Thus, parallel sides are 8 cm and $(8 + 7) = 15$ cm.

4. If one of the parallel sides of a trapezium is 26 cm, find the other parallel side when the area of the trapezium is $729\ \text{cm}^2$ and the distance between the parallel sides is 18 cm.

Solution : One of the parallel sides $(a) = 26$ cm

Area of trapezium $= 729\ \text{cm}^2$

Height $(h) = 18$ cm

$\therefore$ Area of Trapezium $= \frac{1}{2} \times h \times (a + b)$

$$729 = \frac{1}{2} \times 18 \times (26 + b)$$

or $$729 = 9 \times (26 + b)$$

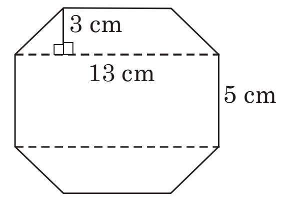
$$\frac{729}{9} = 26 + b$$

$$81 = 26 + b$$

or $$b = 81 - 26$$

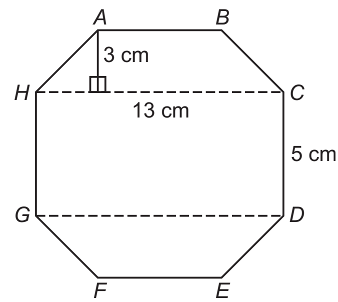
$$b = 55 \text{ cm}$$

So, the other side of trapezium is 55 cm.



5. The top surface of a table is in the shape of a regular octagon as shown in the adjoining figure. Find the area of the octagon.

Solution : The area of octagon $\Rightarrow$ Area of trapezium $ABCH$ + area of trapezium $GDEF$ + Area of rectangle $HCDG$



$\Rightarrow$ $$\frac{1}{2} \times h \times (a+b) + \frac{1}{2} \times h \times (a+b) + l \times b$$

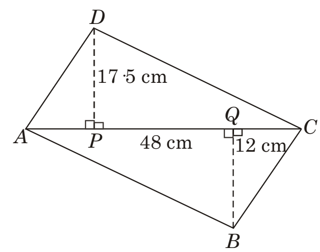
$$= \frac{1}{2} \times 3 \times (13+5) + \frac{1}{2} \times 3 \times (13+5) + 13 \times 5$$

$$= \frac{1}{2} \times 3 \times 18 + \frac{1}{2} \times 3 \times 18 + 65$$

$$= 27 + 27 + 65$$

$$= 119 \text{ cm}^2$$

6. In the adjoining quadrilateral $ABCD$, diagonal $AC = 48$ cm. DP and BQ are perpendiculars to AC, $DP = 17 \cdot 5$ cm and $BQ = 12$ cm. Find the area of quadrilateral $ABCD$.



Solution : $\therefore$ Area of Quadrilateral $ABCD$

= Area of Δ ADC + Area of Δ ABC

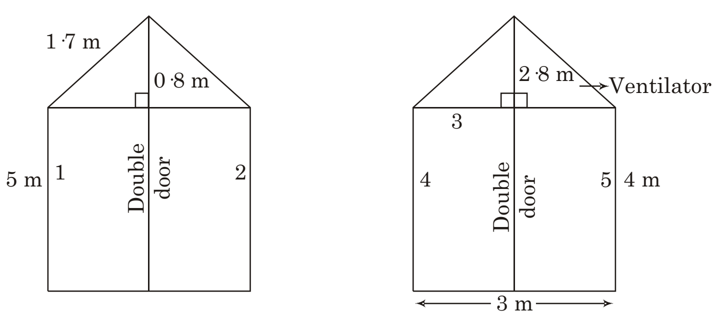
$$= \frac{1}{2} \times b \times h_1 + \frac{1}{2} \times b \times h_2$$

$$= \frac{1}{2} \times 48 \times 17 \cdot 5 + \frac{1}{2} \times 48 \times 12$$

$$= 420 + 288$$

$$= 708 \text{ cm}^2$$

7. Mr. *D*' Souza is renovating his house. He has designed two entrances which are as shown in the figures (a) and (b). Find the area of each section of the two entrances.

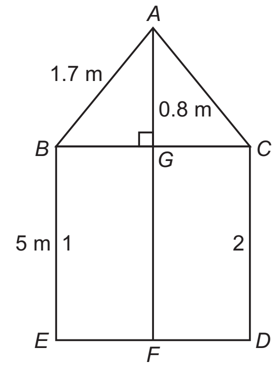


(a) (b)

Solution : In right angled $\Delta\ ABG$,

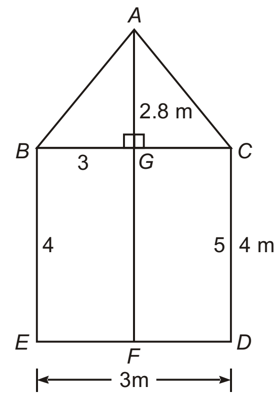
$$(BG)^2 = (AB)^2 - (AG)^2$$
$$= (1\cdot 7)^2 - (0\cdot 8)^2$$
$$= 2\cdot 89 - 0\cdot 64$$
$$(BG)^2 = 2\cdot 25$$

or $BG = 1\cdot 5$ cm



$\therefore$ Area of each section $= \frac{1}{2} \times h \times (BE + AF)$

$$= \frac{1}{2} \times 1\cdot 5 \times (5 + 5\cdot 8)$$
$$= \frac{1}{2} \times 1\cdot 5 \times 10\cdot 8$$
$$= 8\cdot 10 \text{ m}^2$$



Area of III section $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 3 \times 2\cdot 8$$
$$= 4\cdot 2 \text{ m}^2$$

Area of each section (IV and V)

$$= l \times b$$
$$= 4 \times \frac{3}{2}$$
$$= 6 \text{ m}^2$$

Exercise-4

1. What is the total surface area of a cube of side $1\cdot 2$ m?

Solution : Side of cube $= 1\cdot 2$ m

$\therefore$ Total surface area of cube $= 6a^2$

$$= 6 \times (\text{side})^2$$
$$= 6 \times 1\cdot 2 \times 1\cdot 2$$
$$= 8\cdot 64 \text{ m}^2$$

2. Total surface area of a cubical box is 294 cm^2. Find the length of its edge.

Solution : T.S.A. of cubical box $= 294 \text{ cm}^2$

TSA of cube $= 6a^2$

$$294 = 6 \times a^2$$
$$\frac{294}{6} = a^2$$
$$49 = a^2$$

or $a = \sqrt{49}$

$a = 7$ cm

Hence, length of edge = 7 cm

3. A cuboid has total surface area of 160 m^2 and lateral surface area of 120 m^2. Find the area of its base.

Solution : TSA of a cuboid = 160 m^2

LSA of a cuboid = 120 m^2

$$\text{Area of its base} = \frac{1}{2} \times [\text{TSA of cuboid} - \text{LSA of cuboid}]$$

$$= \frac{1}{2}[160 - 120]$$

$$= \frac{1}{2} \times 40 = 20 \text{ m}^2$$

4. The ratio of the curved surface area and the total surface area of a right circular cylinder is 5 : 7. Find the ratio between the height and radius of the cylinder.

Solution : Given :

$$\frac{\text{CSA of cylinder}}{\text{TSA of cylinder}} = \frac{5}{7}$$

$$\frac{2\pi rh}{2\pi r(h+r)} = \frac{5}{7}$$

or $$\frac{h}{h+r} = \frac{5}{7}$$

$$7h = 5h + 5r$$

$$7h - 5h = 5r$$

$$2h = 5r$$

or $$\frac{h}{r} = \frac{5}{2}$$

So, $h : r = 5 : 2$

5. If the total surface area of a cubical box is 864 cm^2, find the edge of the cube.

Solution : TSA of cubical box = 864 cm^2

$$6 \times a^2 = 864$$

or $$a^2 = \frac{864}{6}$$

$$a^2 = 144$$

or $$a = \sqrt{144}$$

$$a = 12 \text{ cm}$$

∴ Edge of the cube is 12 cm

6. A room with a flat roof, a rectangular in shape of breadth 3 m, length $4 \cdot 5$ m and height $3 \cdot 5$ m. It is to be painted inside on the walls and on the ceiling but not on the floor, find the cost of painting at the rate of ₹ 8 per square metre.

Solution : Length (l) = $4 \cdot 5$ m

Breadth $(b) = 3$ m

Height $(h) = 3 \cdot 5$ m

Required surface area = Area of 4 walls + Area of ceiling

$= 2\,(lh + bh) + lb$

$= 2\,(4 \cdot 5 \times 3 \cdot 5 + 3 \times 3 \cdot 5) + 4 \cdot 5 \times 3$

$= 2\,(15 \cdot 75 + 10 \cdot 5) + 13 \cdot 5$

$= 2 \times 26 \cdot 25 + 13 \cdot 5$

$= 52 \cdot 5 + 13 \cdot 5$

$= 66 \cdot 0 \text{ m}^2$

$\therefore$ The cost of painting at the rate ₹ 8 per m^2

$= ₹\, 66 \times 8$

$= ₹\, 528$

7. The outer surface of wooden box of dimensions $75 \text{ cm} \times 60 \text{ cm} \times 40 \text{ cm}$ has to be painted. If the cost of painting 100 cm^2 is ₹15, find the total cost of painting the box.

Solution : Length of wooden box = 75 cm

Breadth of wooden box = 60 cm

Height of wooden box = 40 cm

Required surface Area $= 2\,(lb + bh + hl)$

$= 2\,(75 \times 60 + 60 \times 40 + 40 \times 75)$

$= 2\,(4500 + 2400 + 3000)$

$= 2 \times 9900$

$= 19800 \text{ cm}^2$

The cost of painting $100 \text{ cm}^2 = ₹\,15$

The cost of painting $1 \text{ cm}^2 = ₹\, \dfrac{15}{100}$

The cost of painting $19800 \text{ cm}^2 = ₹\, \dfrac{15}{100} \times 19800$

$= ₹\, 2970$

8. What is the difference between the two boxes shown in the adjoining figure? Is the lateral surface area of both same?

Solution : First figure is cylinder and second figure is cube.

LSA of cylinder $= 2\pi rh$

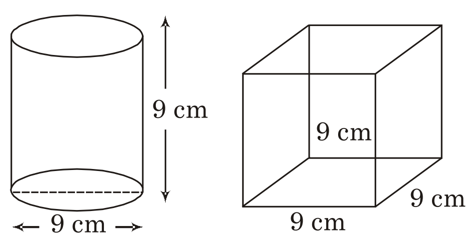
$= 2 \times \pi \times \dfrac{9}{2} \times 9$

$= 81\pi \text{ cm}^2 = 254 \cdot 34 \text{ cm}^2$

LSA of cube $= 4a^2$

$= 4 \times 9 \times 9$

$= 324 \text{ cm}^2$



No, the LSA of both are not same.

9. A closed cylindrical tank of height 8 m and radius $3 \cdot 5$ m is to be made from a metal sheet. Find the cost of the metal sheet required at the rate of ₹130 per m^2.

Solution : Height of cylinder = 8 m

Radius of cylinder = $3 \cdot 5$ m

Required surface area of closed cylinder = $2\pi r\ (h + r)$

$$= 2 \times \frac{22}{7} \times 3 \cdot 5\ (8 + 3 \cdot 5)$$

$$= 2 \times 22 \times 0 \cdot 5 \times 11 \cdot 5$$

$$= 253 \text{ m}^2$$

∴ The cost of the metal sheet at the rate of ₹130 per m^2

$$= ₹\, 253 \times 130$$

$$= ₹\, 32890$$

10. The curved surface area of a hollow cylinder is 4224 cm^2, it is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of the rectangular sheet.

Solution : CSA of a hollow cylinder = 4224 cm^2

Width of rectangular sheet = 33 cm

∴ Height of cylinder = 33 cm

∴ $2\pi rh = 4224$

$$2\pi r \times 33 = 4224$$

$$2\pi r = \frac{4224}{33}$$

$$2\pi r = 128 \text{ cm}$$

∴ Length of rectangular sheet = 128 cm

Now, Perimeter of rectangular sheet = $2\ (l + b)$

$$= 2\ (128 + 33)$$

$$= 2 \times 161$$

$$= 322 \text{ cm}$$

11. Three cubes each of side 6 cm area joined end to end. Find the surface area of resulting cuboid.

Solution : Three cubes each of side 6 cm are joined end to end, then we get a cuboid having :

Length $(l) = 6 + 6 + 6 = 18$ cm

Breadth $(b) = 6$ cm

Height $(h) = 6$ cm

∴ Required surface area of resulting cuboid = $2 \times (lb + bh + hl)$

$$= 2 \times (18 \times 6 + 6 \times 6 + 6 \times 18)$$

$$= 2 \times (108 + 36 + 108)$$

$$= 2 \times 252$$

$$= 504 \text{ cm}^2$$

12. A suitcase with measure 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suit cases?

Solution : Length of suitcase = 80 cm

Breadth of suitcase = 48 cm

Height of suitcase = 24 cm

$\therefore$ Total surface area of suitcase = $2\,(lb + bh + hl)$

$= 2\,(80 \times 48 + 48 \times 24 + 24 \times 80)$

$= 2\,(3840 + 1152 + 1920)$

$= 2 \times 6912 = 13824\ \text{cm}^2$

Required cloth to cover 100 such suitcases with width 96 cm = $\frac{13824}{90\ \text{cm}} \times 100\ \text{cm}^2$

$= 14400\ \text{cm}$

$= 144\ \text{m}$

13. A class room is 7 m long, 6 m broad and 4 m high. The doors and windows occupy an area of 7 m^2. Find the cost of whitewashing the walls and roof at the rate of ₹15 per m^2.

Solution : Length of classroom = 7 m

Breadth of classroom = 6 m

Height of classroom = 4 m

$\therefore$ Required surface area = Area of 4 walls + Area of roof

$= 2\,(bh + hl) + lb$

$= 2\,(6 \times 4 + 4 \times 7) + 7 \times 6$

$= 2\,(24 + 28) + 42$

$= 146$

Area without doors and windows = $146 - 7 = 139\ \text{m}^2$

$\therefore$ The cost of whitewashing the walls and roof at the rate of ₹15 per m^2 = ₹139×15 = ₹2,085

Exercise-5

1. Calculate the volume of a cube whose edge is :

(a) 5 m

Solution : Edge of cube = 5 m

Volume of cube = $(\text{edge})^3$

$= (5\ \text{m})^3$

$= 5 \times 5 \times 5\ \text{m}^3$

$= 125\ \text{m}^3$

(b) 20 cm

Solution : Edge of cube = 20 cm

Volume of cube = $(\text{edge})^3$

$= (20\ \text{cm})^3$

$= 20 \times 20 \times 20\ \text{cm}^3$

$= 8000\ \text{cm}^3$

2. If the total surface of a cube is 3750 cm^2, find its volume.

Solution : TSA of cube = $3750\ \text{cm}^2$

$$\text{TSA of cube} = 6 \times a^2$$
$$3750 = 6 \times a^2$$
$$\frac{3750}{6} = a^2$$
$$625 = a^2$$
$$a^2 = 25^2$$
$$a = 25 \text{ cm}$$
$$\text{Now, volume of cube} = (\text{edge})^3$$
$$= (25 \text{ cm})^3$$
$$= 15625 \text{ cm}^3$$

3. What is the volume of a cubical tank of water of side $1 \cdot 2$ m?
Solution : The side of water tank $= 1 \cdot 2$ m

$$\text{Volume of cubical tank} = (\text{side})^3$$
$$= (1 \cdot 2 \text{ m})^3$$
$$= 1 \cdot 728 \text{ m}^3$$

4. If the edge of a cube is doubled,
(a) By how many times will its surface area increase?
Solution : edge of cube $= 2 \times a$

$$\text{Surface area} = 6 \times (\text{edge})^2$$
$$= 6 \times (2a)^2$$
$$= 6 \times 4a^2$$
$$= 4 \times (6a^2)$$

So, the area will increase 4 times.
(b) By how many times will its volume increase?
Solution : Edge of cube $= 2 \times a$

$$\text{Volume} = (\text{edge})^3$$
$$= (2a)^3$$
$$= 8a^3$$
$$= 8 \times a^3$$

So, the volume will increase 8 times.

5. A cylindrical tank has a capcacity of 6160 m^3. Find its depth if the diameter is 28 m.
Solution : Volume of cylindrical tank $= 6160 \text{ m}^3$

$$\text{Volume of cylinder} = \pi r^2 h$$
$$6160 = \frac{22}{7} \times \frac{28}{2} \times \frac{28}{2} \times h$$
$$h = \frac{6160 \times 7 \times 2 \times 2}{22 \times 28 \times 28}$$
$$h = 10 \text{ m}$$

So, the depth of the tank is 10 m.

6. Find the number of coins, each of radius $0 \cdot 75$ cm and thickness $0 \cdot 2$ cm, that should be melted to make a right circular cylinder of height 8 cm and base of radius 3 cm.

Solution : Radius of coin $(r) = 0 \cdot 75$ cm

$$\text{Height of coin } (h) = 0 \cdot 2 \text{ cm}$$

$$\text{Volume of coin} = \pi r^2 h$$

$$= \frac{22}{7} \times 0 \cdot 75 \times 0 \cdot 75 \times 0 \cdot 2$$

$$\text{Radius of cylinder} = 3 \text{ cm}$$

$$\text{Height of cylinder} = 8 \text{ cm}$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 3 \times 3 \times 8$$

$$\therefore \quad \text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\frac{22}{7} \times 3 \times 3 \times 8}{\frac{22}{7} \times 0 \cdot 75 \times 0 \cdot 75 \times 0 \cdot 2}$$

$$= 640 \text{ coins}$$

7. The ratio of surface area of two cubes is 16 : 49. Find the ratio of their volumes.

Solution : Given ratio of surface areas of two cubes $= 16 : 49$

$$\frac{4a^2}{4A^2} = \frac{16}{49}$$

or $$\frac{a^2}{A^2} = \frac{16}{49}$$

or $$\frac{a}{A} = \frac{4}{7}$$

Now, ratio of their volumes $= \dfrac{a^3}{A^3}$

$$= \left(\frac{4}{7}\right)^3$$

$$= \frac{64}{343}$$

$$= 64 : 343$$

8. A rectangular piece of paper of width 30 cm and length 77 cm is rolled along its width to form a cylinder, what is the volume of the cylinder so formed?

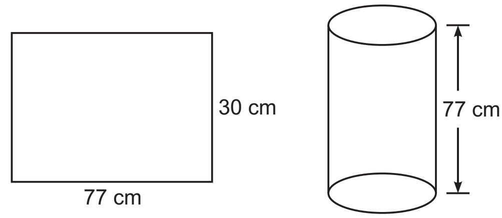
Solution : $\because$ Length of rectangle = Circumference of base

$$30 = 2\pi r$$

$$30 = 2 \times \frac{22}{7} \times r$$

or $$r = \frac{30 \times 7}{2 \times 22}$$

$\therefore$ $$r = \frac{105}{22}$$



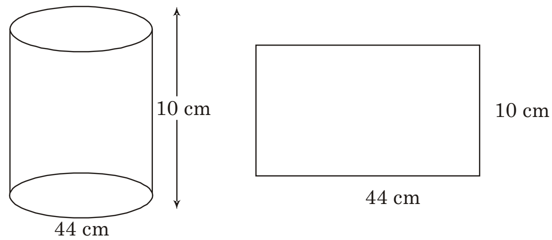
Height of cylinder = 77 cm

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{105}{22} \times \frac{105}{22} \times 77$$

$$= 5512 \cdot 5 \text{ cm}^3$$

9. A rectangular piece of paper 44 cm long and 10 cm broad is rolled along the length to form a cylinder. What is the radius of the base?



Solution : Circumference of base $= 2\pi r$

$$44 = 2 \times \frac{22}{7} \times r$$

or $$r = \frac{44 \times 7}{2 \times 22}$$

$\therefore$ $$r = 7$$

So, the radius of base is 7 cm.

10. Find the volume of a right circular cylinder whose curved surface area is 2640 cm^2 and circumference of the base is 66 cm.

Solution : CSA of cylinder $= 2640 \text{ cm}^2$

$$2\pi rh = 2640 \quad \ldots \quad \text{(i)}$$

Circumference of base $= 2\pi r = 66$ cm

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$$

Frrom eqn. (i), $66 \times h = 2640$

$$h = \frac{2640}{66}$$

$$= 40 \text{ cm}$$

$\therefore$ Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40$$

$$= 13860 \text{ cm}^3$$

11. Water flows into a tank from a tap whose inner radius is $0 \cdot 75$ cm. If the water flows at the rate of 7 m/s, how many litres of water flow into the tank in half an hour ?

Solution : Speed of flowing water = 7 m/s

= 700 cm/sec

Time = half an hour $= 30 \times 60$ sec.

$$\text{Radius} = 0 \cdot 75 \text{ cm}$$

$$\text{Volume} = \pi r^2 h$$

$$= 3 \cdot 14 \times 0 \cdot 75 \times 0 \cdot 75 \times 700 \times 30 \times 60$$

$$= 2225475 \text{ cm}^3$$

$$= \frac{2225475}{1000} \text{ litre} = 2225 \cdot 475$$

$$= 2225 \cdot 5 \text{ litre}$$

12. The dimensions of a hall are 150 m × 85 m × 12 m. How many persons can sit in the hall if each person requires 50 m^3 of air?

Solution : No. of persons that can sit $= \dfrac{\text{Volume of hall}}{\text{Air required by each person}}$

$$= \frac{150 \times 85 \times 12 \text{ m}^3}{50 \text{ m}^3}$$

$$= 3 \times 85 \times 12$$

$$= 3060 \text{ persons}$$

13. The length of a room is double its breadth. The area of the four walls is 192 m^2. If the height of the room is 4 m, find its volume.

Solution : Let breadth of room be x

Then length of room will be $2x$.

Height of room = 4 m

Area of four walls = 192 m^2.

$$2(bh + hl) = 192$$

$$2h(b + l) = 192$$

$$2 \times 4(x + 2x) = 192$$

$$8 \times 3x = 192$$

$$24x = 192$$

or $$x = \frac{192}{24}$$

$$x = 8 \text{ m}$$

∴ Length $= 2x = 2 \times 8 = 16$ m

Breadth $= x = 8$ m

Now, volume of room $= l \times b \times h$

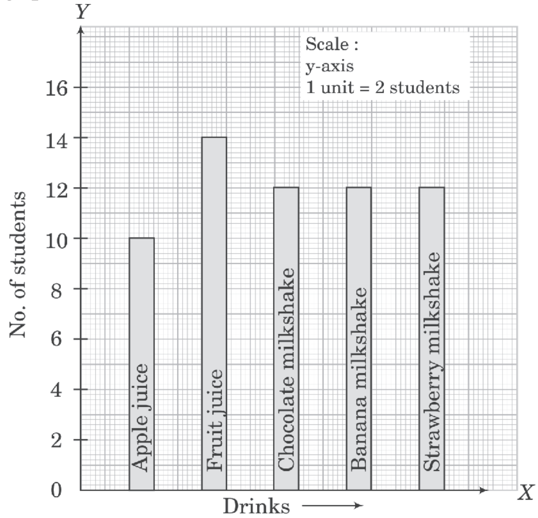
$$= 16 \times 8 \times 4$$

$$= 512 \text{ m}^3$$

Chapter-16 : Data Handling

Exercise-1

1. The bar graph shown in the following figure shows the favourite drinks of students in class 6. Answer the following questions :



(a) How many kids chose chocolate milkshake as their favourite drink?

(b) How many more kids voted for fruit juice than strawberry milkshake?

(c) If six more kids voted for banana milkshake, would it be the drink with most votes?

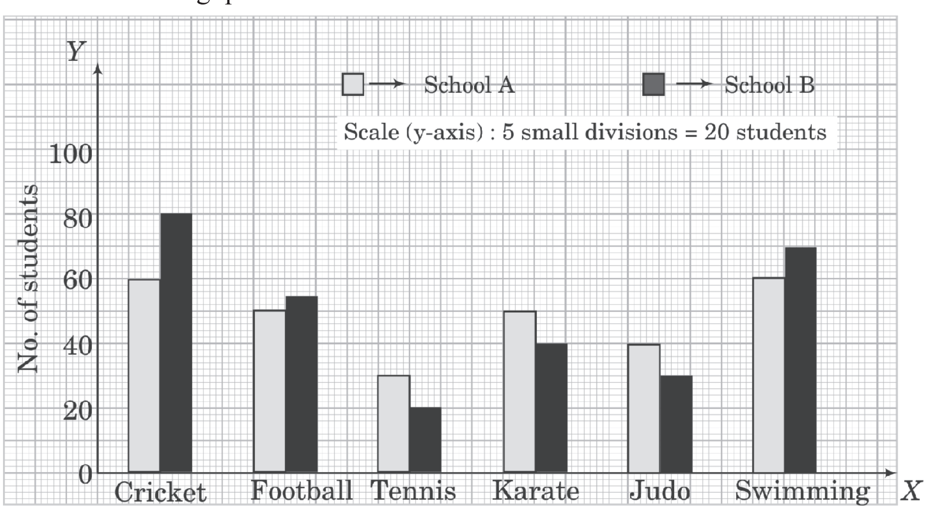
Solution : (a) 12

(b) $14 - 12 = 2$

(c) Total No. of kids voted for banana milkshake $= 12 + 6 = 18$

Yes, now it would be the drink with most votes.

2. Given below is a double bar graph showing hobbies of the students of two schools. Read the graph and answer the following questions.



(a) In which school is cricket more popular?
(b) In which school is karate more popular?
(c) In which school is tennis least popular?
(d) What percentage of students of each school prefer swimming?

Solution : (a) In school B Cricket is more popular.
(b) In school A Karate is more popular.
(c) In school B Tennis is least popular.
(d) % of school A prefer swimming $= \frac{60}{300} \times 100\%$
$= 20\%$

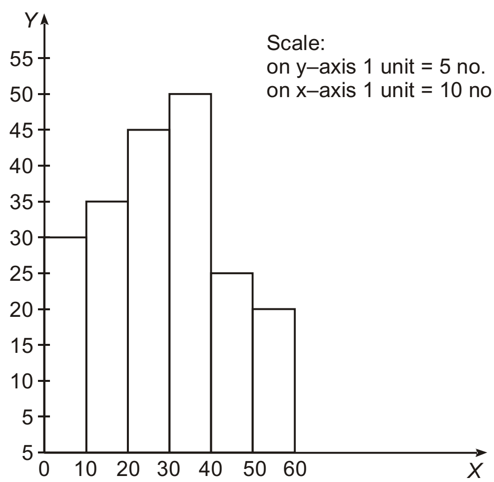
% of school B prefer swimming $= \frac{70}{300} \times 100\%$
$= 23 \cdot 33\%$

[Here, sum of all students of school A and B is 300]

3. Draw a histogram for the following data :

Class interval	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Frequency	30	35	48	53	25	15

Solution :



4. A student wants to find the number of hours spent by her on the computer each week. The student collects data for 40 weeks as follows :

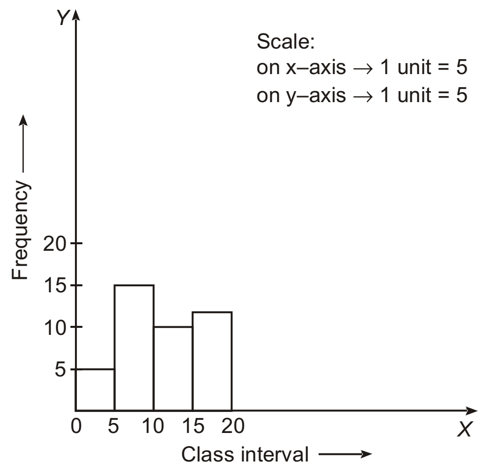
0	10	20	6	6	2	10	5	5	10
0	6	5	20	20	10	10	20	1	6
10	15	15	10	20	6	6	6	5	10
15	15	20	20	10	5	5	5	6	2

(a) Construct a frequency distribution table for the above data using $0-5$, $5-10$ etc.
(b) Represent the above data in the form of histogram.

Solution : (a) Frequency distribution Table :

Class interval	Tally marks	Frequency
0 – 5	𝍸	5
5 – 10	𝍸 𝍸 𝍸	15
10 – 15	𝍸 ||||	9
15 – 20	𝍸 𝍸 |	11

(b)



5. Marks obtained by 5 students of class 8 in half yearly and final examination 2010 in mathematics (out of 50) are given below. Represent this data by means of double bar graph and answer the given questions :

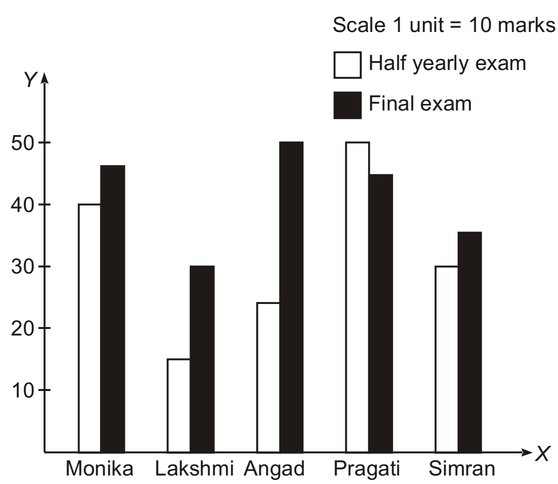
Students	Monika	Laksmi	Angad	Pragati	Simran
Half yearly exam	40	15	25	48	29
Final exam	42	28	48	46	33

(a) Do you see any improvement in exams?

(b) Who has not done better than before in final exam?

Solution : (a) Yes, except Pragati all students have improved in their exams.

(b) Pragati.



Exercise-2

1. The number of students admitted in different faculties of a college are given below :

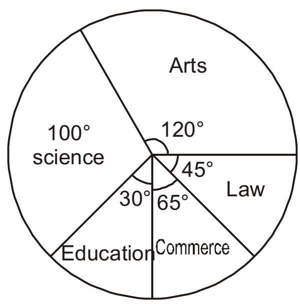
Faculty	Science	Arts	Commerce	Law	Education	Total
No. of Students	1000	1200	650	450	300	3600

Draw a pie chart to represent the above informations.

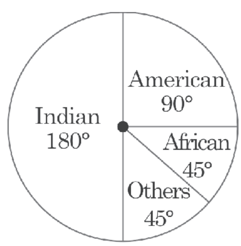
Solution : Total No. of students admitted in different faculties is 3600, converted into component parts of 360°.

Faculty	**No. of students**	**Measure of central angle**
Science	1000	$\frac{1000}{3600} \times 360° = 100°$
Arts	1200	$\frac{1200}{3600} \times 360° = 120°$
Commerce	650	$\frac{650}{3600} \times 360° = 65°$
Law	450	$\frac{450}{3600} \times 360° = 45°$
Education	300	$\frac{300}{3600} \times 360° = 30°$

Now, pie chart is :



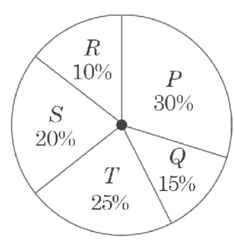
2. The following pie chart depicts the percentage of students nation wide in a school. What is the percentage of Indian students nationwide ?



Solution : The percentage of Indian students $= \frac{180°}{360°} \times 100\%$

$= 50°$

3. The adjoining pie chart shows the percentage of buyers of five brands of shampoo P, Q, T, S and R.



(a) Which is the most popular brand of shampoo?

(b) Which is the least popular brand of shampoo?

(c) What is the central angle of the sector of shampoo S?

(d) What is the number of persons purchasing shampoo P if the number of persons purchasing shampoo T is 270?

Solution : (a) P is the most popular brand of shampoo.

(b) R is the last popular brand of shampoo.

(c) Central angle of the sector of shampoo $S = \frac{20\%}{100\%} \times 360°$

$= 72°$

(d) Given : the no. of persons purchasing shampoo T is 270.

i.e. 25% of total persons = 270

$\therefore$ Total persons $= 270 \times \frac{100}{25} = 1080$ persons

$\therefore$ No. of persons purchasinhg shampoo $P = 1080 \times \frac{30}{100}$

$= 324$ persons

4. A survey was conducted on the favourite pet animal of 72 people. The findings are as follows. Represent this data by a pie chart.

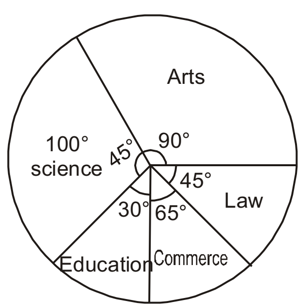
Pet animal	Dog	Cat	Rabbit	Fish	Birds
No. of persons	27	6	9	18	12

Solution : Total no. of persons = 72

Now, we calculate the degree of angles to draw pie chart :

Pet animal	**No. of persons**	**Measure of central angle**
Dog	27	$\frac{27}{72} \times 360° = 135°$
Cat	6	$\frac{6}{72} \times 360° = 30°$
Rabbit	9	$\frac{9}{72} \times 360° = 45°$
Fist	18	$\frac{18}{72} \times 360° = 90°$
Birds	12	$\frac{12}{72} \times 360° = 60°$

Pie chart is :



5. Students of class 8 were asked about their favourite cartoon characters. The following data was collected.

Mickey Mouse	Batman	Doraemon	Dora	Donald Duck
120	69	51	6	24

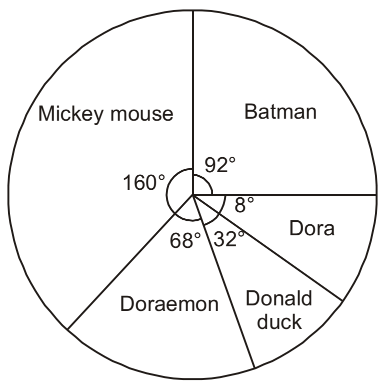
Represent the above data using a pie chart.

Solution : Total students = 270

Now, we calculate the degree of angles to draw pie chart.

Cartoon characters	No. of students	Measure of central angle
Mickey Mouse	120	$\frac{120}{270} \times 360° = 160°$
Batman	69	$\frac{69}{270} \times 360° = 92°$
Doraemon	51	$\frac{51}{270} \times 360° = 68°$
Dora	6	$\frac{6}{270} \times 360° = 8°$
Donald Duck	24	$\frac{24}{270} \times 360° = 32°$

Pie chart is :



Exercise-3

1. Find the probability of getting an ace from a well shuffled pack of 52 cards.

Solution : Total cards = 52

No. of aces = 4

$\therefore$ Probability of getting an ace $(P) = \frac{4}{52}$

$= \frac{1}{13}$

2. A die is thrown once. Find the probability of getting (a) a 3

(b) An even number.

Solution : Total possible outcomes = 1, 2, 3, 4, 5, 6

(a) Probability of getting a 3 $(P) = \frac{1}{6}$

(b) Probability of an even no. $(P) = \frac{3}{6}$

$= \frac{1}{2}$

3. A glass jar contains 2 red, 3 blue, 5 yellow and 1 green marble. A single marble is chosen at random write the sample space and also find out the probability of each outcome.

Solution : Total No. of marbles = 11

No. of red marbles = 2

No. of blue marbles = 3

No. of yellow marbles = 3

No. of green marble = 1

∴ Sample space = $\{R, R, B, B, B, Y, Y, Y, Y, Y, G\}$

Now, probability of red marble outcomes = $\frac{2}{11}$

Probability of blue marble outcomes = $\frac{3}{11}$

Probability of yellow marble outcomes = $\frac{5}{11}$

Probability of green marble outcomes = $\frac{1}{11}$

4. Two dice are thrown simultaneously. Find the probability of getting :

(a) A sum less than 5 (b) A doublet of even numbers

(c) A prime number as the sum (d) A multiple of 3 as the sum.

Solution : Elementary events associated to the random experiment of throwing two dice are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)...... (6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) ...... (6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) ...... (6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) ...... (6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) ...... (6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)...... (6)

∴ Total no. of elementary events = $6 \times 6 = 36$

(a) Possible outcomes (sum of less than 5)

= (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1) = 6

∴ Probability of getting (Sum of less than 5)

$= \frac{6}{36} = \frac{1}{6}$

(b) Possible outcomes (doublet of even numbers)

= (2, 2), (4, 4), (6, 6), = 3

∴ Probability $= \frac{3}{36} = \frac{1}{12}$

(c) Possible outcomes (Prime no. as the sum) = (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5) = 15

Probability = $\frac{15}{36} = \frac{5}{12}$

(d) Possible outcomes (multiple of 3 as the sum) = (1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6) = 12

Probability = $\frac{12}{36} = \frac{1}{3}$

5. In a closed box, there are two blue balls, three red balls and five green balls (the balls are identical). If you have to put your hand in the box and take out one ball, without seeing, what is the probability of the ball,

(a) Being blue (b) Being red (c) Being green

Solution : Total no. of outcomes = 10

No. of blue balls = 2

No. of red balls = 3

No. of green balls = 5

(a) P (blue ball) = $\frac{2}{10} = \frac{1}{5}$

(b) P (red ball) = $\frac{3}{10}$

(c) P (green ball) = $\frac{5}{10} = \frac{1}{2}$

6. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken at random from the box. Find the probability that the bulb is not defective.

Solution : Total outcomes = 400

No. of defective bulbs = 15

No. of non-defective bulbs = 385

$\therefore$ Probability of getting a non-defective bulb = $\frac{385}{400} = \frac{77}{80}$

7. A bag contains slips on which English alphabets $a, b, c, \ldots x, y, z$ are written. If one slip is taken out, what is the probability that the slip contains

(a) A vowel (b) A consonant (c) The letter '*m*'

Solution : Total possible outcomes = 26

(a) No. of vowels = a, e, i, o, u = 5

$\therefore$ P (a vowel) = $\frac{5}{26}$

(b) No. of consonants = 21

P (a consonant) = $\frac{21}{26}$

(c) P (letter '*m*') = $\frac{1}{26}$

8. A coin is tossed on two successive times. Find the probability of getting :

(a) Exactly one head (b) No heads (c) Both heads

Solution : Possible outcomes = 4 (Both heads, both tails, one head and one tail, one tail and one head)

(a) Probability of getting exactly one head $P = \frac{2}{4}$

$= \frac{1}{2}$

(b) Probability of getting No heads (both tails) $= \frac{1}{4}$

(c) Probability of getting both heads $= \frac{1}{4}$

9. A card is drawn from a deck of 52 cards. What is the probability of :

(a) Getting an ace of clubs (b) Getting an 8 (c) Getting a heart

Solution : Total possible outcomes = 52

(a) No. of an ace = 1

$\therefore$ Probability of getting an ace $= \frac{1}{52}$

(b) No. of eights in the deck = 4

$\therefore$ Probability of getting an eight $= \frac{4}{52} = \frac{1}{13}$

(c) No. of hearts in the deck = 13

$\therefore$ Probability of getting a heart $= \frac{13}{52} = \frac{1}{4}$

Chapter-17 : Graphs

Exercise-1

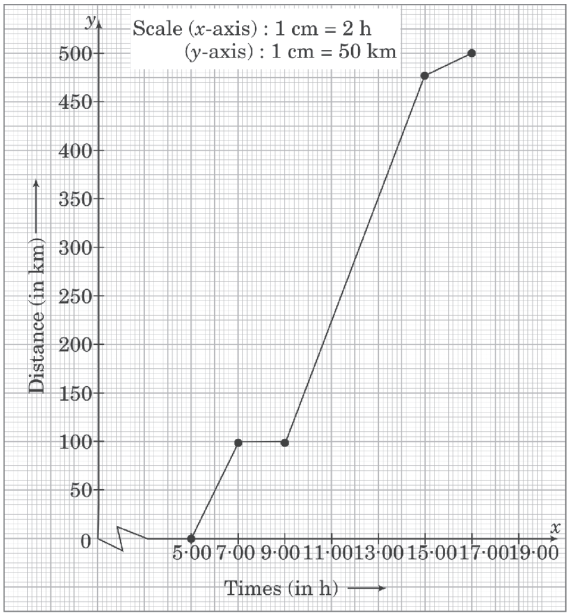
1. The distance covered by a car in travelling from city A to city B, 500 km away, is shown in the figure given below. Study the graph and answer the following questions :

(a) What is the distance travelled in the first 2 hours ?

(b) What is the distance covered by the car from 9 : 00 to 15 : 00 hours ?

(c) Did the car stop at any point during the journey? Give reason for your answer.

(d) What is the total travelling time of the journey?



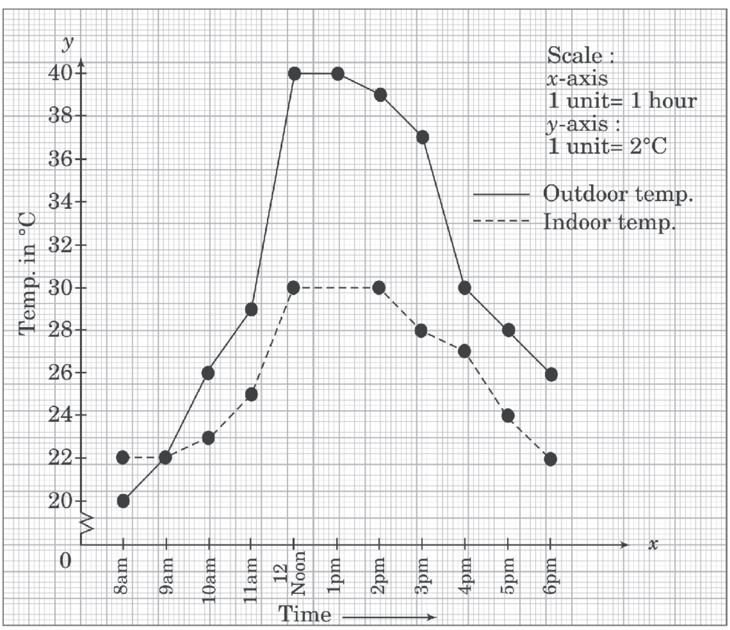
Solution : (a) Distance in first 2 hours $= 0 + 100 = 100$ km

(b) Distance covered from $9 \cdot 00$ to 15 hrs. $= 475 - 100 = 375$ km

(c) Between 7 : 00 to 9 : 00, the distance of the car from the starting and end is the same (100 km), the car was stationary.

(d) Total time $= 17 \cdot 00 - 5 \cdot 00 = 12$ hours

2. The double line graph in the following figure shows indoor and outdoor temperatures during a particular day.



Study the graph and answer the following questions :

(a) What are the indoor and outdoor temperatures at 10 am?

(b) When is the temperature difference highest?

(c) Find the difference between indoor and outdoor temperatures at 2 pm. ?

(d) At what time of the day is the outdoor temperature less than the indoor temperature?

(e) During what time did the indoor temperature remain constant?

Solution : (a) Indoor temperature at 10 am = 23 °C

Outdoor temperature at 10 am = 26 °C

(b) The highest temperature difference is at 12 noon. $(40-30=10\ °C)$

(c) Difference between indoor and outdoor temperatures at 2 pm = $(39-30)$ °C = 9 °C

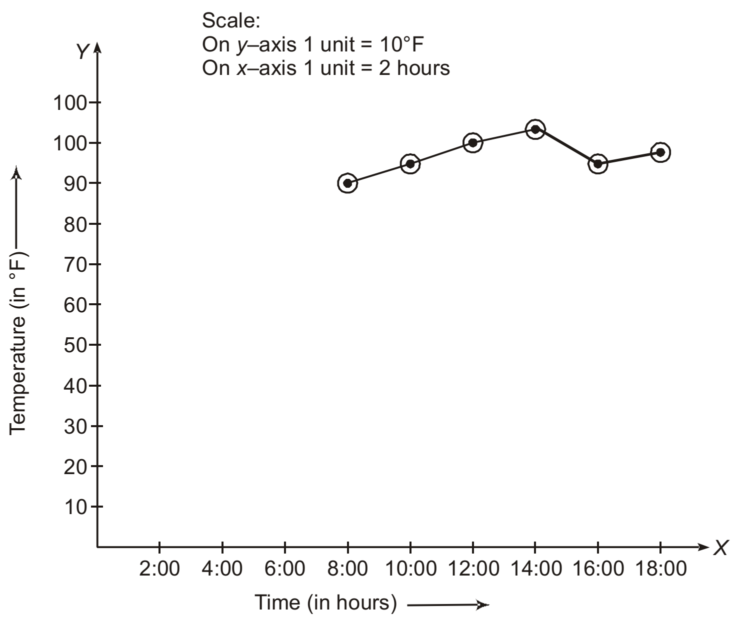
(d) The outdoor temperature less than the indoor temperature at 8 am $(22-20)=2°$ C

(e) 12 noon to 2 pm (between 12 noon and 2 pm we get a straight line *i. e.* temperature is constant).

3. The following is the body temperature of a patient recorded at regular intervals of time. Draw a temperature-time graph for the same.

Time (in hours)	8·00	10·00	12·00	14·00	16·00	18·00
Temperature (in °F)	98	99	101	103	100	98

Solution :

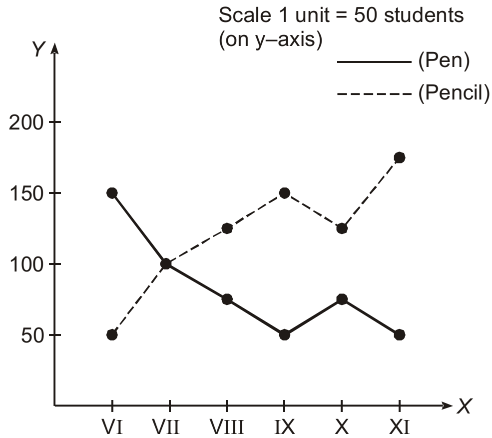


4. The number of students who prefer to use pen or pencil to take down notes in five different classes is as follows :

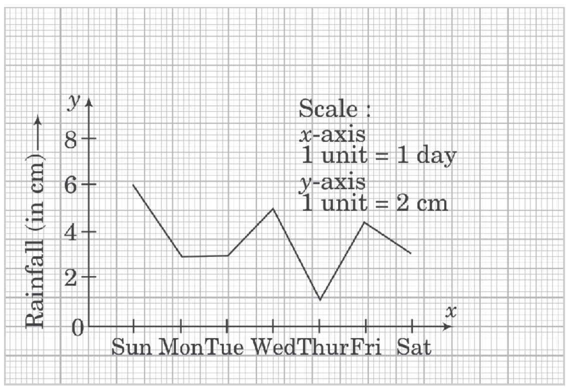
Preferences	VI	VII	VIII	IX	X	XI
No. of students who prefer pen	150	100	75	50	75	25
No. of students who prefer pencil	50	100	125	150	125	175

Using the above data, draw a line graph.

Solution :



5. The line graph given below shows the rainfall in centimetres in New Delhi during a week.



Study the graph and answer the following questions :

(a) Find the amount of rainfall on Wednesday.

(b) On which day of the week was the rainfall minimum?

(c) Find the decrease or increase in rainfall from Thursday to Friday.

(d) Find the average rainfall during the week.

(e) On which days was the amount of rainfall the same?

Solution : (a) 5 cm

(b) Thursday

(c) $(4 \cdot 5 - 1 \cdot 0) = 3 \cdot 5$ cm

(d) Average rainfall $= \dfrac{6+3+3+5+1+4\cdot5+3}{7}$

$= \dfrac{25\cdot5}{7}$

$= 3\cdot64$ cm

(e) On Monday, Tuesday and Saturday amount of rainfall was the same.

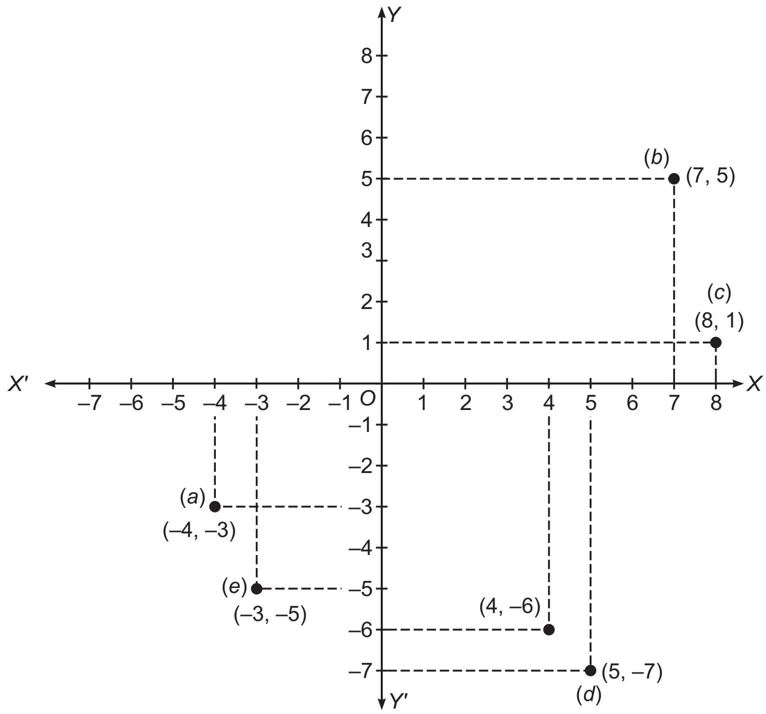
Exercise-2

1. Plot the following points on a graph paper :

(a) (– 4, – 3) (b) (7, 5) (c) (8, 1)

(d) (5, – 7) (e) (–3, – 5) (f) (4, – 6)

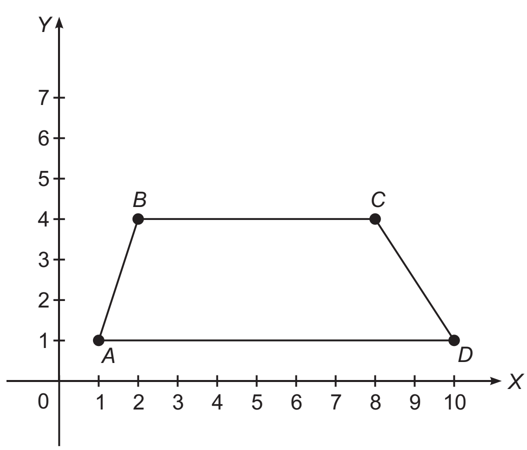
Solution :



2. Plot the following points and join them :

(a) *A* (1, 1), *B* (2, 4), *C* (8, 4) and *D* (10, 1) and what figure do you obtain ?

Solution : Trapezium.



3. In which quadrant, will the following points lie? Also, write their distances from the y-axis.

(a) $(1, 2)$ (b) $(-3, 4)$ (c) $(-4, -5)$ (d) $(10, 0)$ (e) $(5, -2)$

Solution :

(a) $(1, 2) = (+, +)$ First quadrant

(b) $(-3, 4) = (-, +)$ Second quadrant

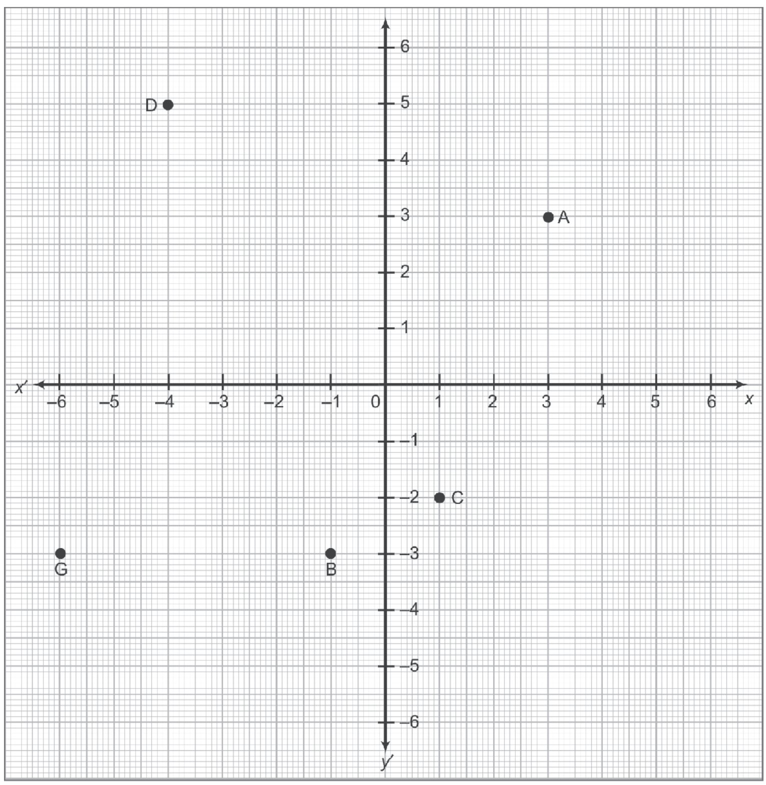
(c) $(-4, -5) = (-, -)$ Third quadrant

(d) $(10, 0) = (+, +)$ First quadrant

(e) $(5, -2) = (+, -)$ Fourth quadrant

4. From the following figure, choose the points that indicate the location of the points given below :

(a) $(1, -2)$ (b) $(-4, 5)$ (c) $(-1, -3)$



Solution : Point A is $(3, 3)$

Point B is $(-1, -3)$

Point C is $(1, -2)$

Point D is $(-4, 5)$

Point G is $(-6, -3)$

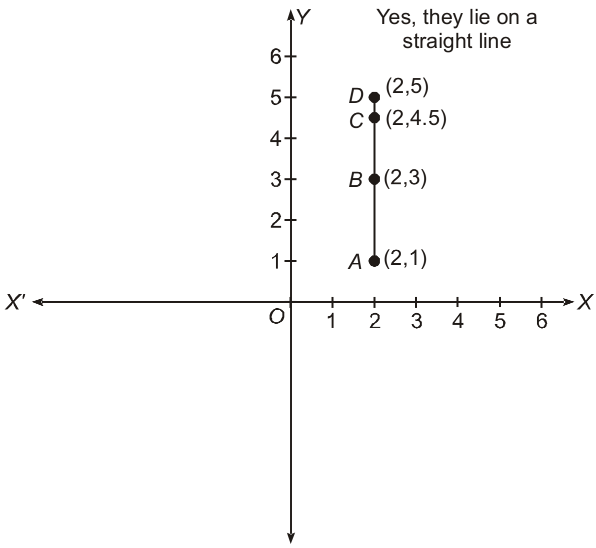
5. Plot the following points on a graph sheet and verify if they lie on a straight line :

(a) $A\ (2, 1)$; $B\ (2, 3)$; $C\ (2, 4\cdot5)$; $D\ (2, 5)$

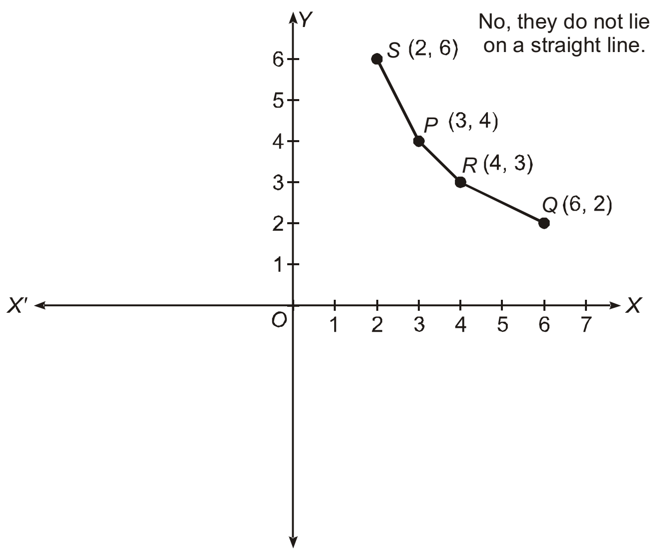
(b) $P\ (3, 4)$; $Q\ (6, 2)$; $R\ (4, 3)$; $S\ (2, 6)$

(c) $L\ (1, 3)$; $M\ (2, 6)$; $N\ (3, 9)$; $P\ (4, 12)$

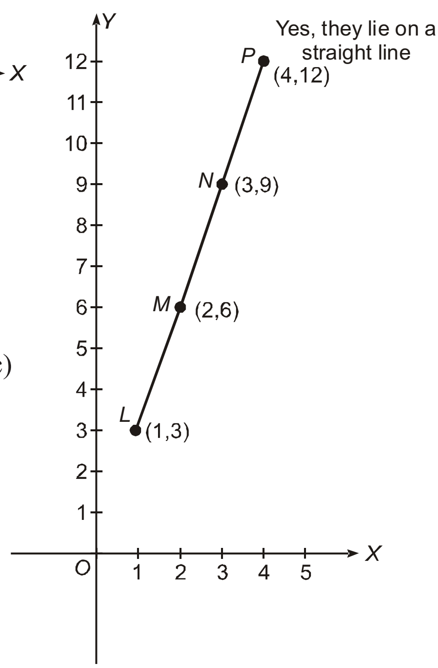
Solution : (a)



(b)



(c)

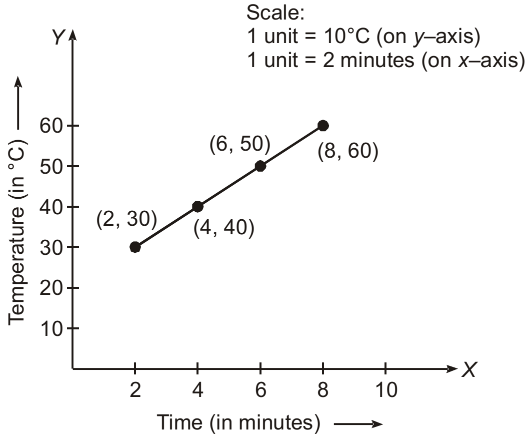


Exercise-3

1. Temperature (T) in °C of a metal ball in time (t) minutes is given in the following table. Plot a line graph for the same :

Temperature (in °C)	30	40	50	60
Time (in minute)	2	4	6	8

Solution :



2. Draw a graph for the following values in the table (taking a suitable scale) showing distance travelled by a bus starting from the depot at 4.00 a.m.

Time (a. m.)	$5\cdot00$	$6\cdot00$	$7\cdot00$	$8\cdot00$	$9\cdot00$
Distance (km)	60	120	180	240	300

Answer the following questions :

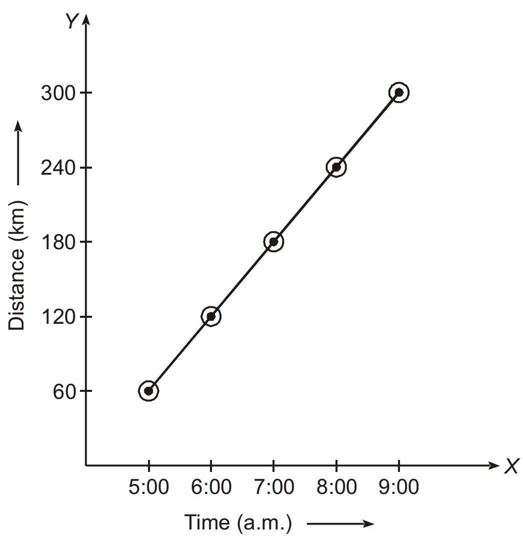
(a) How much distance did the bus cover between $6\cdot30$ and $7\cdot30$ hours?

(b) What would be the time when the bus had covered 200 km?

Solution :

(a) $(210-150)$ km $= 60$ km.

(b) 7 : 20 a.m.



3. The following table shows the values of the number of notebooks and their cost. Represent this data by a graph and answer the following questions :

No. of Notebooks	1	4	6	8
Cost (₹)	$12 \cdot 50$	50	75	100

(a) What will be the cost of 5 notebooks?

(b) How many notebooks can be purchased for ₹ 25?

Solution : (a) Cost of 5 notebooks $= ₹\ (50 + 12 \cdot 50)$

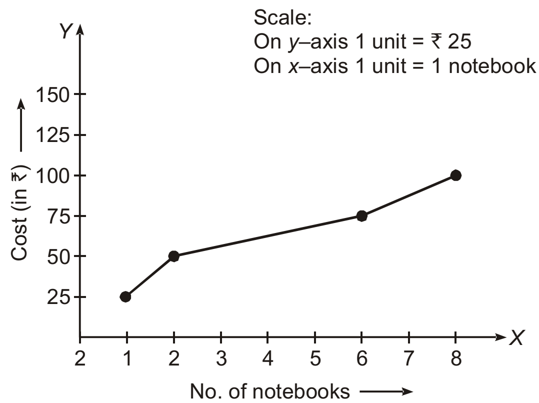
$= ₹\ 62 \cdot 50$

(b) No. of notebooks for $₹\ 12 \cdot 50 = 1$

No. of notebooks for $₹\ 25 = \frac{1}{12 \cdot 50} \times 25$

$= 2$

notebooks



4. A taxi driver hired a taxi for 6 hours. Plot a speed-distance graph for the following data and answer the questions given below using the graph :

Speed (in km/h)	20	25	30	40	50
Distance (in km)	120	150	180	240	300

(a) Find the distance covered if he is travelling at a speed of 45 km/h.

(b) Find the speed of the taxi if he travels a distance of 450 km.

Solution : (a) Distance covered at 20 km/h = 120 km

∴ Distance covered at 45 kn/h

$= \frac{120}{20} \times 45$

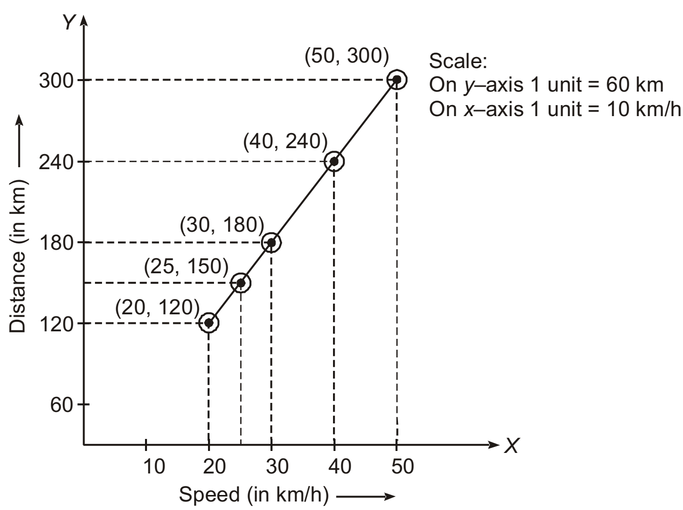
$= 270$ km

(b) Speed in 120 km

$= 20$ km /h

∴ Speed in 450 km

$= \frac{20}{120} \times 450 = 75$ km/h



5. A carpenter is putting beading around square tables. The following data gives the length of beading required

and the side of the square table :

Side (in m)	2	2·5	3	4
Length of beading required (in m)	8	10	12	16

Plot a graph for the above data and use it to answer the following questions :

(a) What is the length of the beading required for a square table with side 5 m?

(b) What is the cost of putting beading around a square table of side 6 m at the rate of ₹110 per m ?

(c) Find the side of the square table if the length of the beading used is 20 m.

Solution : (a) Length of beading with side 2 m = 8 m

Length of beading with side 5 m $= \frac{8}{2} \times 5$

$= 20$ m

(b) Length of beading with side 2 m = 8 m

Length of beading with side 6 m $= \frac{8}{2} \times 6$

$= 24$ m

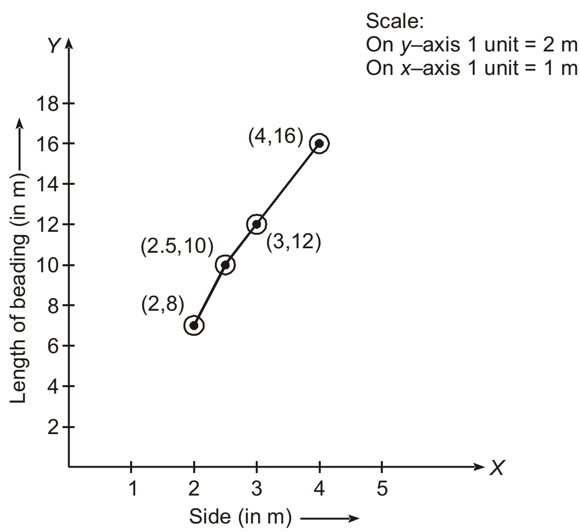
Required cost = ₹ (110×24)

= ₹ 2640

(c) Side of square table if length of beading is 8 m = 2 m

Side of square table if length of beading is 20 m $= \frac{2}{8} \times 20$

$= 5$ m



✿